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SUBMANIFOLDS SATISFYING THE CONDITION $K(X, Y)\cdot K=0$

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Introduction.

In 1968, Simons [7] obtained a formula giving the Laplacian of the square of length of the second fundamental tensor and applied it to the study of minimal hypersurfaces of a sphere. Nomizu and Smyth [6] applied a formula of Simons' type to the study of hypersurfaces with constant mean curvature and with non-negative sectional curvature in a Euclidean space or in a sphere. Chern, Do Carmo and Kobayashi [2] also applied Simons' formula to the study of minimal submanifolds of a sphere (see also Chern [1]). Recently, Yano and Ishihara [10] have applied a formula of Simons' type to the study of submanifolds of higher codimension with parallel mean curvature vector and with locally trivial normal bundle in a Euclidean space or in a sphere. On the other hand, Nomizu [5] studied hypresurfaces of a Euclidean space, which satisfy the condition $K(X, Y)\cdot K=0$ for all tangent vectors X and Y, K being the curvature tensor. Tanno [8], Tanno and Takahashi [9] studied hypersurfaces of a Euclidean space or of a sphere, which satisfy the condition $K(X, Y)\cdot S=0$ for all tangent vectors X and Y, S being the Ricci tensor (see also Kenmotsu [4]).

In the present paper, we shall, applying a formula of Simons' type, study submanifolds satisfying the condition $K(X, Y)\cdot K=0$ and having parallel mean curvature vector, non-negative Ricci curvature and locally trivial normal bundle in a space of constant curvature. We shall also study submanifolds with parallel second fundamental tensor and with locally trivial normal bundle in a Euclidean space or in a sphere. The main results are stated in Theorems 3. 3, 3. 4, 3. 5 and 3. 6.

§ 1. Preliminaries.

Let M^m be an m-dimensional Riemannian manifold of class C^∞ with metric tensor G, whose components are G_{ji} with respect to local coordinates $\{\xi^h\}$. Let M^n be an n-dimensional connected submanifold of class C^∞ differentiably immersed in M^m $(1<n<m)$ and suppose that the local expression of the submanifold M^n is

$$\xi^h=\xi^h(\eta^a), \tag{1. 1}$$

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