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EXTREMAL PROPERTIES OF QUASIHARMONIC FORMS AND FUNCTIONS

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The purpose of the present paper is to deduce extremal properties of differential forms φ satisfying the differential equations

$$\delta d T\varphi + PT\varphi = 0$$

or

$$\delta d T\varphi + d\delta S\varphi = 0$$

on a Riemannian space. For suitable choices of the operators T and S and the nonnegative function P we obtain, in a unified manner, extremal properties of harmonic, semiharmonic, cosemiharmonic, quasiharmonic, and coquasiharmonic forms, and harmonic, P-harmonic, quasiharmonic, and P-quasiharmonic functions.

§ 1. Fundamentals.

1. Let $F(u, v)$ be a bilinear form on a real linear space V, and set $F(u)=F(u, u)$. Consider a subset H of V such that for each $h\in H$, $F(h)\geqq 0$. For a fixed $u\in V$ set $v=u+h$ for $h\in H$. We characterize u by an extremal property.

The function u minimizes the functional $\{F(v)-F(h, u)-F(u, h)\}$ *and the minumum is* $F(u)$:

$$F(v)-F(h, u)-F(u, h)=F(u)+F(h). \tag{1}$$

If F is an inner product, set

$$F(u, v)=[u, v], \qquad |||u^2|||=[u, u]. \tag{2}$$

The function u minimizes the functional $\{|||v|||^2-2[h, u]\}$ *among all* $v\in V$ *with* $h=v-u$, *and the minimum is* $|||u|||^2$:

$$|||v|||^2-2[h, u]=|||u|||^2+|||h|||^2. \tag{3}$$

We specialize further.

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