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A REMARK ON UNIPOTENT GROUPS OF CHARACTERISTIC $p>0$

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Borel and Springer [1] deal with a unipotent group U defined over a field of prime characteristic p and investigate the conditions of the existence of the one dimensional subgroup to which a given element of the Lie algebra $L(U)$ of U is tangent. They use the lemma (9.15) (ii) (p. 493) in the proof of the last theorem (9.16) (ii) (p. 495). In this report we shall show that this lemma is not correct (cf. § 2). But a modification of it does not disturb the truth of the theorem. Moreover we want to show that the theorem (9.16) (iii) in [1] is still true under a weaker assumption (cf. § 1). The notations in this report are the same as in [1].

§ 1. In this section we give a modification of the lemma (9.15) (ii) in [1]. It is given as a corollary of the following lemma. The form of the weight in the assumption is changed from $(p^i+p^j)a$ to p^ia+p^jb $(a, b \in \Psi)$. The proof proceeds similarly to that given in [1].

LEMMA. *Let U be a unipotent k-group and T a k-torus which acts k-morphically on U. Let N be a connected central k-subgroup of U, stable under T, such that U/N is commutative. Let U_1, W and W_1 be subgroups of U containing N, stable under T, such that U_1 and W_1 are normal subgroups of U and W, respectively, and such that U/U_1 and W/W_1 are isomorphic to $\boldsymbol{G}_a$. Put $\Phi(T, U/U_1)=\{a\}$ and $\Phi(T, W/W_1)=\{b\}$. If $\Phi(T, N)$ does not contain any element of the form p^ia+p^jb $(i, j \geqq 0)$ and the commutator groups (U, W_1) and (U_1, W) are trivial, then (U, W) is also trivial.*

Proof. Let

$$\alpha: \quad U \times W \to N$$

be the commutator map, sending (x, y) to $x \cdot y \cdot x^{-1} \cdot y^{-1}$. Using that (U, W_1) and (U_1, W) are trivial, we see that this induces a T-equivariant morphism

$$\alpha': \quad U/U_1 \times W/W_1 \to N.$$

By Chevalley ([2] Exp. 9, lemme 2, p. 1), we may find a composition series

$$N=N_0 \supset N_1 \supset \cdots \supset N_q=\{e\}$$

of connected subgroups, stable under T, such that successive quotients are isomorphic

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