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SUBMANIFOLDS OF MANIFOLDS WITH AN f-STRUCTURE

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Let M^n be an n-dimensional C^∞ manifold and f a tensor of type (1, 1) such that

$$f^3+f=0,$$

and the rank of f is constant, say r, on M^n. We then say that M^n has an *f-structure* of rank r (Cf. [4]). The rank r of f is necessarily even and it is known that if r is maximal, then f is an almost complex structure on M^n if n is even or an almost contact structure on M^n if n is odd (Cf. [4]). Yano and Ishihara [5] have shown that if M^n is an almost complex manifold then a submanifold of M^n satisfying a certain property possesses a natural f-structure. In particular, Tashiro [3] has shown that if the submanifold is a hypersurface then the induced f-structure has maximal rank (i.e. is almost contact). On the other hand, the present author and Prof. D. E. Blair [1] have shown that a hypersurface of an almost contact manifold possesses a natural f-structure, which may not have maximal rank.

The purpose of this paper is to show that if M^n has an f-structure then a submanifold of M^n satisfying the condition of Yano and Ishihara possesses a natural f-structure. In § 3 we examine the meaning of this condition in the special case where the submanifold is a hypersurface. § 4 is devoted to a study of the integrability of the induced f-structure.

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§ 1. Preliminaries.

Let M^n be a given n-dimensional C^∞ manifold. Let f be a given f-structure on M^n of rank r. Then the tensors l and m, where $l=-f^2$ and $m=f^2+I$, are complementary projection operators, i.e.

$$l^2=l, \qquad m^2=m, \tag{1.1}$$
$$l+m=I, \qquad lm=ml=0.$$

Here I denotes the identity operator. Thus, there exist in M^n complementary distributions L and M corresponding to l and m respectively. The dimension of L is r and the dimension of M is $n-r$. If $n=2k$ and $r=2k$ we denote f by J and

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