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ON INFINITESIMAL DEFORMATIONS OF CLOSED HYPERSURFACES

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§ 1. Introduction.

In the present paper we study the effect of infinitesimal deformations of (1) a closed orientable hypersurface in an orientable Riemannian manifold and (2) a closed hypersurface in a Euclidean space on some integrals.

Let M be an $(n+1)$-dimensional orientable Riemannian manifold and M' be a closed orientable hypersurface in M whose equations are given by

$$x^h = x^h(u^a)$$

in local coordinates. We use indices h, i, j, k for M and a, b, c, d for M', hence h, i, j, k run over the range $\{1, \cdots, n+1\}$ and a, b, c, d over the range $\{\dot{1}, \cdots, \dot{n}\}$. As usual $B_a{}^h$ means $\partial_a x^h$ where $\partial_a = \partial/\partial u^a$. $g_{ba} = B_b{}^i B_a{}^h g_{ih} = B_{ba}^{ih} g_{ih}$ are the components of the first fundamental tensor of M'. The unit normal vector is denoted by N^h and the reciprocal of the matrix $(B_a{}^h, N^h)$ by $(B^a{}_h, N_h)$. ∇ means the Van der Waerden-Bortolotti differential operator, hence $\nabla_b B_a{}^h = h_{ba} N^h$, $\nabla_b N^h = -h_b{}^a B_a{}^h$ where $h_b{}^a = h_{bc} g^{ca}$. h_{ba} are the components of the second fundamental tensor of M'.

§ 2. Infinitesimal deformations.

Let $\mathcal{M}'$ be a set of hypersurfaces $M'(t)$, $0 \leqq t < \varepsilon$, where ε is a sufficiently small positive number and $M'(0) = M'$. We assume that the local coordinates of the points of $M'(t)$ are given by

$$x^h = x^h(u^a, t)$$

in M. We also assume that $x^h(u^a, t)$ are C^∞ functions and the mapping $\varphi(t)$: $M'(0) \to M'(t)$ induced by

$$x^h(u^a, 0) \to x^h(u^a, t) \tag{2.1}$$

is diffeomorphic, u^a being local coordinates of $M'(t)$ in $U \cap M'(t)$ for some neighborhood U of M and for all $t \in [0, \varepsilon)$. $\varphi(t)$ is a deformation of M'.

We define $\xi^h(u^a)$ by

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