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PROLONGATIONS OF HYPERSURFACES TO TANGENT BUNDLES

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Introduction.

The prolongations of tensor fields and connections to tangent bundles have been recently discussed in [1], [2], [3] and [4]. Ishihara, Kobayashi and Yano defined and studied prolongations called complete, vertical and horizontal lifts of tensor fields and connections. In this paper we introduce the notion of prolongations of surfaces to tangent bundle, which seems to be a natural one, and develop the theory of surfaces prolonged to the tangent bundle with respect to the metric tensor which is the complete lift of the metric tensor of the original manifold. We shall define in § 3 the vertical and the complete lifts of the vector fields defined along the surface, and choose two kinds of lifts of the normal vector field of the surface as vector fields normal to the prolonged surface.

We shall recall in § 4 some formulas for surfaces for the later use and give, for prolonged surfaces, some of fundamental formulas containing the so-called second fundamental tensors in § 5. In the last section the equations of Gauss, of Weingarten, and the so-called structure equations, those of Gauss, of Codazzi and of Ricci, for the prolonged surface are formulated in the form of lifts of the corresponding equations of the surface given in the base space.

§ 1. Notations.

For any differentiable manifold N, we denote by $T(N)$ its tangent bundle with the projection π_N: $T(N)\to N$, and by $T_p(N)$ its tangent space at a point p of N. $\mathcal{T}^r_s(N)$ is the space of tensor fields of class C^∞ and of type (r, s), i.e., of contravariant degree r and covariant degree s in N. An element of $\mathcal{T}^0_0(N)$ is a C^∞-function defined on N. We denote by $\mathcal{T}(N)$ the tensor algebra on N, i.e., $\mathcal{T}(N) =\sum_{r,s}\mathcal{T}^r_s(N)$.

Let M be an n-dimensional differentiable manifold and V a coordinate neighborhood in M and (x^i) certain local coordinates defined in V. We introduce a system of coordinates (x^i, y^i) in $\pi_M^{-1}(V)$ such that (y^i) are cartesian coordinates in each tangent space $T_p(M)$, p being an arbitrary point of V, with respect to the natural frame $(\partial/\partial x^i)$ of local coordinates (x^i). We call (x^i, y^i) the coordinates induced in $\pi_M^{-1}(V)$ from (x^i), or simply the induced coordinates in $\pi_M^{-1}(V)$. (cf. [3], [4]).

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