ON CONVOLUTION OF POWER SERIES

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1. In their recent paper [3] Pólya and Schoenberg have stated a conjecture on power series mapping a circle onto a convex domain. Let $\mathfrak{K}$ be the class of analytic functions which are regular in the unit circle and map it univalently onto convex domains. They then state

CONJECTURE. *If both functions*

$$f(z)=\sum_{n=1}^{\infty} a_n z^n \quad \textit{and} \quad g(z)=\sum_{n=1}^{\infty} b_n z^n$$

belong to $\mathfrak{K}$, *also the function defined by*

$$h(z)=\sum_{n=1}^{\infty} a_n b_n z^n$$

belongs to $\mathfrak{K}$.

In the present note, considering a related class, it will be shown that an analogous proposition is affirmatively verified. Let $\mathfrak{R}$ be the class of analytic functions $\Phi(z)$ regular in the unit circle and characterized by

$$\Re\Phi(z)>0 \quad \text{for} \quad |z|<1 \quad \text{and} \quad \Phi(0)=1.$$

We will then establish the following

THEOREM 1. *If both functions*

$$f(z)=1+2\sum_{n=1}^{\infty} a_n z^n \quad \textit{and} \quad g(z)=1+2\sum_{n=1}^{\infty} b_n z^n$$

belong to $\mathfrak{R}$, *also the function defined by*

$$h(z)=1+2\sum_{n=1}^{\infty} a_n b_n z^n$$

belongs to $\mathfrak{R}$.

Proof. Based on the integral representation valid for the class $\mathfrak{R}$, we may put

$$g(z)=\int_{-\pi}^{\pi} \frac{e^{i\theta}+z}{e^{i\theta}-z} d\mu(\theta)$$

where $\mu(\theta)$ is a real-valued function defined for $-\pi<\theta\leqq\pi$ which is increasing and has the total variation equal to unity; cf. e.g. [1]. Consequently,

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