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THE RANK OF AN f-STRUCTURE

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§ 1. Introduction

In [2], K. Yano introduced the notion of an f-structure on a manifold. Specifically, on the manifold M^n one has a tensor field f of type (1,1), i. e. a homomorphism from the tangent bundle of M into itself, satisfying:

$$f^3+f=0.$$

Throughout the literature, it is standard to then suppose f has the same rank, r say, at each point, and one then says that M has an f structure of rank r.

The purpose of this note is to prove the rather surprising observation that the extra assumption is unnecessary.

PROPOSITION: *If f is a tensor field of type* (1,1) *on M satisfying $f^3+f=0$, then the function from M to the integers assigning to x the rank of $f(x)$ is continuous. In particular, the rank of f is automatically constant on the components of M.*

This result is actually a special case of results of Vanžura [1], but is not emphasized there. It seems of adequate significance to justify emphasis.

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§ 2. Proof

Let $\mathrm{Hom}(\tau(M), \tau(M))$ be the bundle of homomorphisms of the tangent bundle of M into itself, i. e. the bundle of tensors of type (1,1).

LEMMA: *The set of $f\in\mathrm{Hom}(\tau(M), \tau(M))$ of rank greater than or equal to k is open.*

Proof: Locally $\mathrm{Hom}(\tau(M), \tau(M))$ is $U\times\mathrm{Hom}(R^n, R^n)$, and this is the open set defined by the nonvanishing of the determinant of some $k\times k$ minor.

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