Y. KUBOTA
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A REMARK ON THE THIRD COEFFICIENT OF MEROMORPHIC UNIVALENT FUNCTIONS

By Yoshihisa Kubota

1. Let G be a domain on the z-sphere containing the origin and let $S(G)$ denote the family of functions $f(z)$ regular and univalent in G with expansion at the origin

$$f(z)=z+\sum_{n=2}^{\infty}a_nz^n.$$

Let D be a domain on the z-sphere containing the point at infinity and let $\Sigma'(D)$ denote the family of functions $f(z)$ meromorphic and univalent in D with expansion at the point at infinity

$$f(z)=z+\sum_{n=1}^{\infty}b_nz^{-n}.$$

The following problem was considered by Schaeffer and Spencer [5]. Let $\mathfrak{G}$ be the set of domains onto which E, the unit circle, is mapped by functions belonging to $S(E)$. For each domain G belonging to $\mathfrak{G}$ we write

$$\alpha_n(G)=\sup_{f\in S(G)}|a_n| \qquad (n=2, 3, \cdots).$$

Find the precise values

$$\gamma_n=\inf_{G\in\mathfrak{G}}\alpha_n(G) \qquad (n=2, 3, \cdots)$$

and

$$\Gamma_n=\sup_{G\in\mathfrak{G}}\alpha_n(G) \qquad (n=2, 3, \cdots).$$

Schaeffer and Spencer showed that $\gamma_n=\alpha_n(E)$ and that if the Bieberbach conjecture is true, then $\Gamma_n=4^{n-1}$.

In this paper we consider a similar problem for meromorphic univalent functions. Let $\mathfrak{D}$ be the set of domains onto which $\tilde{E}$, the exterior of the unit circle, is mapped by functions belonging to $\Sigma'(\tilde{E})$. For each domain D belonging to $\mathfrak{D}$ we write

$$\beta_n(D)=\sup_{f\in\Sigma'(D)}|b_n| \qquad (n=1, 2, \cdots).$$

Further we write

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