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ON LEVEL CURVES OF GREEN'S FUNCTIONS

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1. Let D be a general plane domain on which a Green's function $g(z, z_0)$ with pole at z_0, $z_0 \in D$, exists. It is known that in a sufficiently small neighborhood of the pole each level curve of $g(z, z_0)$ consists of a single analytic Jordan curve and that, as t decreases in $g(z, z_0)=t$, the corresponding level curves may split up into a finite or infinite number of components. More specifically, depending on t, each level curve consists of a single component or of a collection of components, each such component being either a closed curve or an open arc.

The set of those level curves of $g(z, z_0)$ which contain at least one component which is an open arc tending in at least one sense toward a critical point of $g(z, z_0)$ is clearly countable. However, there is not much known about the particular set of level curves of $g(z, z_0)$ which contain as components open arcs tending instead toward irregular boundary points. The purpose of the present paper is to prove that almost all level curves of $g(z, z_0)$ consist of components which are closed curves, where "almost all" is understood with respect to a natural linear measure on the set of all level curves of $g(z, z_0)$. The specific approach taken is from the point of view of Function Theory.

2. Since Green's functions are invariant under conformal mappings it suffices to concentrate in our study on a general plane domain D containing the point at infinity and the corresponding Green's function $g(z, \infty)$ with pole at the point at infinity. The pointset $\left\{z \in D \,|\, g(z, \infty)=t, \left(\frac{\partial g}{\partial x}, \frac{\partial g}{\partial y}\right) \neq (0, 0),\ t \text{ some positive number and } z=x+iy\right\}$ is called a level curve or the t-level curve for $g(z, \infty)$, even if it consists of many curves. Henceforth such a t-level curve will be denoted by c_t. The sense on a level curve c_t will be taken so that at each point of its components a sufficiently small left-hand neighborhood at the point meets only points z at which $g(z, \infty)<t$ and a sufficiently small right-hand neighborhood meets only points z where $g(z, \infty)>t$. The term Green's line or orthogonal trajectory will be used to denote a maximal open arc on D which is orthogonal to the level curves c_t passing through each of its points. A boundary point z_0 of D for which every neighborhood contains a closed sub-

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