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INVARIANT CLOSED GEODESICS UNDER ISOMETRIES OF PRIME POWER ORDER

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§ 0. Introduction

Let M be a Riemannian manifold and h an isometry. A geodesic $\gamma: \boldsymbol{R} \to M$ is called to be invariant under h (or h-invariant) if there exists some number $\theta \geqq 0$ such that $h(\gamma(t))=\gamma(t+\theta)$ for all $t \in \boldsymbol{R}$. Let $C^{\circ}(M, h)$ be the topological space of continuous curves $\sigma: [0, 1] \to M$ satisfying $h(\sigma(0))=\sigma(1)$ with the compact open topology. Two geodesics $\gamma_1, \gamma_2: \boldsymbol{R} \to M$ are called to be geometrically distinct if $\gamma_1(\boldsymbol{R}) \neq \gamma_2(\boldsymbol{R})$. The following is a well-known result on the existence of closed geodesics obtained by Gromoll and Meyer [3].

THEOREM. (*Gromoll-Meyer*). *Let M be a simply connected compact Riemannian manifold. If the sequence of Betti numbers for the space $C^{\circ}(M, id.)$ is not bounded, then there exist infinitely many (geometrically distinct) closed geodesics in M.*

The above theorem gives us the following problem of existence on invariant geodesics under isometries.

Problem. For each fixed isometry h, are there infinitely many h-invariant geodesics in M if the sequence of Betti numbers for the space $C^{\circ}(M, h)$ is not bounded?

This problem was solved positively for involutive isometries by Grove [6] and was solved positively for isometries of prime order by the author [9]. The purpose of this paper is to show that it is also true for isometries of prime power order. Grove claimed first that he could prove the following main theorem. Soon after the author proved it independently and pointed out that Grove's proof was incomplete.

MAIN THEOREM. *Let M be a compact simply connected Riemannian manifold and f an isometry of prime power order. Then there exist infinitely many (geometrically distinct) f-invariant closed geodesics in M if the sequence of Betti numbers for the space $C^{\circ}(M, f)$ is not bounded.*

§ 1. Preliminaries.

Let $(M, \langle\ ,\ \rangle)$ be a compact Riemannian manifold of dimension $n+1$ and g

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