M. KOBAYASHI AND S. TSUCHIYA
KŌDAI MATH. SEM. REP.
29 (1977), 103—119

HOLOMORPHIC ISOMORPHISM WHICH PRESERVES CERTAIN HOLOMORPHIC SECTIONAL CURVATURE

BY MINORU KOBAYASHI AND SUSUMU TSUCHIYA

1. Introduction. Let (M, g) and $(\bar{M}, \bar{g})$ be two Riemannian manifolds. Denote the corresponding sectional curvatures by K and $\bar{K}$ respectively. A diffeomorphism f from M to $\bar{M}$ will be said to be curvature preserving if and only if for every $p \in M$ and for every 2-plane σ in the tangent space $T_p(M)$ to M, we have

$$K(\sigma)=\bar{K}(f_*\sigma).$$

It is natural to ask whether a curvature preserving diffeomorphism is isometric or not. The answer to this question was first given by R.S. Kulkarni as follows;

THEOREM ([2]). *If M is an analytic Riemannian manifold with dimension $\geqq 4$, then a curvature preserving diffeomorphism $f: M \mapsto \bar{M}$ is an isometry except in the case that both M and $\bar{M}$ have the same constant curvature.*

In the case where both of (M, g) and $(\bar{M}, \bar{g})$ are Kaehlerian manifolds, we may expect that a holomorphic sectional curvature preserving diffeomorphism is a isometry. Indeed he proved

THEOREM ([4]). *Let M and $\bar{M}$ be connected Kaehlerian manifolds with corresponding holomorphic sectional curvature functions H and $\bar{H}$ respectively. Suppose that $\dim M \geqq 2$ and there exists a diffeomorphism $f: M \mapsto \bar{M}$ such that $f^*\bar{H}=H$. Then either $H=\bar{H}=const.$ or f is holomorphic or anti-holomorphic isometry.*

On the other hand, in our previous paper ([5]), we defined the θ-holomorphic sectional curvature and the τ-bisectional curvature and showed that the constancy of the holomorphic sectional curvature is equivalent to that of the θ-holomorphic sectional curvature or to that of the holomorphic τ-bisectional curvature. It is then quite natural to ask whether a θ-holomorphic sectional curvature preserving or a holomorphic τ-bisectional curvature preserving diffeomorphism is isometric or not. Concerning this problems, we shall prove the following two theorems. We shall define in Section 3 what are called θ-holomorphically isocurved Kaehlerian manifolds and what are called τ-bisectionally isocurved Kaehlerian manifolds.

Received June 26, 1976.