

## ON SOME OPERATIONS IN THE BORDISM THEORY WITH SINGULARITIES

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### § 1. Introduction.

In [11], Sullivan constructed the bordism theory with singularities. Let  $S$  be a closed manifold. Then in this theory " $\overline{W}$  is a closed manifold with singularities of type  $S$ " means

$$\overline{W} = W \cup (\text{cone } S) \times L \text{ (along boundary)}$$

where  $W$  is a manifold with  $\partial W \cong L \times S$  and  $L$  is a closed manifold. Then we can define a bordism operation  $Q_S$  by  $Q_S(\overline{W}) = L$ . In this paper, we study this operation.

Throughout this paper, let manifolds be stable almost complex manifolds. For finite complex  $X$  the bordism group  $MU(S)_*(X)$  is defined by the bordism classes of maps from closed manifolds with singularities of type  $S$  to  $X$ .

By taking the stratification of singularities, Sullivan also defined the theory when singularity is a set of manifolds and proved that the ordinary mod  $p$  homology theory is the bordism theory with singularities of type  $(p, x_1, x_2, \dots)$  i.e.  $H_*(X; \mathbb{Z}_p) \cong MU(p, x_1, x_2, \dots)_*(X)$  where  $x_i$  denote  $2i$ -dimensional ring generators of  $MU_*(S^0) = MU_*$ . By using the Quillen's theorem [9], we shall show  $H_*(X; \mathbb{Z}_p) \otimes \mathbb{Z}_p[\dots, x_i, \dots] \cong MU(p, v_1, v_2, \dots)_*(X)$  where  $v_i$  denote  $x_{p^i-1}$  which are Milnor manifolds for a fixed prime  $p$ .

Let  $I_n$  be the set  $(p, v_1, \dots, v_n)$  and let  $MU(I_n)$  be the spectrum of the theory  $MU(I_n)_*(-)$ . We denote by  $Q'_i$  the Spanier-Whitehead dual operation of  $Q_{v_i}$ . Our main results of this paper are as follows

**THEOREM 3.4.** *If  $y \in H^*(X; \mathbb{Z}_p)$  then  $\lambda Q'_i(y) = Q_i(y)$  for some  $\lambda \neq 0 \in \mathbb{Z}_p$ , where  $Q_i$  is the Milnor exterior operation.*

**THEOREM 4.1.**  *$MU(I_n)^*(MU(I_n)) \cong MU^*/I_n \otimes_{MU^*} MU^*(MU) \otimes \Lambda[Q''_0, \dots, Q''_n]$ . where  $Q''_i$  are cohomology operations which satisfies  $Q''_i(y) = Q'_i(y)$  for each finite complex  $X$  and each element  $y \in MU(I_n)^*(X)$ .*

In this paper we always assume that  $p$  is a fixed prime number, (co)homology theories are reduced theories and  $X$  is a finite complex.

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