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ALGEBRAIC FUNCTIONS*

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PREFACE.

The whole thesis consists of three chapters. In chapter I, we deal with the structure of Rational Functions at various places of the Riemann - Surface of an Algebraic Function and deduce some new results. It also serves as an introduction to the rest of the chapters.

The second chapter consists of three parts; the first one gives three theorems concerning the structure of the branching of the Riemann - Surface of the Fundamental Equation. The second one deals with the investigation of the differential coefficient of an Algebraic Function. This produces a result which is an improvement over the result already published by Beatty. The third part is merely to show how to extend these results to all algebraically closed fields of characteristic zero.

Chapter III consists of two main parts. The first part is a proof of the Riemann - Roch Theorem, and the second is its applications. A new method of proof for the Riemann - Roch Theorem based mostly on the ideas of analysis is given. In doing so important new theorems are introduced. In the second part it is demonstrated that some of the well-known results in the Algebraic Function Theory are easily deduced by the application of the new method.

References to various chapters are given at the end of the thesis in the Bibliography.

CHAPTER I.

THEOREMS ON THE STRUCTURE OF RATIONAL ALGEBRAIC FUNCTIONS.

1. Let $f(z,u)\equiv f_0 u^n+f_1 u^{n-1}+\cdots+f_n = 0$ be an irreducible algebraic equation (f_s are rational functions in z with coefficients in the field of complex numbers k), defining the field of rational functions $k(z,u)$. If $a, b \in k$ is a solution of

$$f(z,u)=0$$

then there exists a formal power series, solution of

$$f(z,u)=0$$

in the form

$$(a)\quad \begin{cases} z-a = t^{r} \\ u-b = t^{\sigma}(a_\sigma + a_{\sigma+1}t+\cdots) \end{cases} \qquad a_\sigma \neq 0$$

where r, σ are integers and $\sigma > 0$. Such a pair of functions (a) is called a place-representation of the Riemann-Surface of the Algebraic Function.

2. Value of a Rational Function at a Place.

Let a Rational Function $R(z,u) \in k(z,u)$. Let π be given by a place-representation (a). In virtue of the substitution (a), we have at π,

$$R(z,u)= t^{\rho}(a_0 + a_1 t + \cdots)$$

where

$$a_0 \neq 0.$$

If $\rho > 0$, then $R(\pi)$ is said to have zero of order ρ at the place π, and $\rho < 0$ is said to have a pole of order $-\rho$ at the place, and $\rho = 0$, $R(\pi)$ is regular.

3. At every place of the Riemann-Surface of the Algebraic Function, any rational function $R(z,u)$ has either a pole, or a zero of some definite order or is regular in the sense of paragraph 2. Also every rational function η has a unique divisor except for a constant. This can be represented symbolically as

$$\eta \backsim \frac{P_1^{\gamma_1}\cdots P_t^{\gamma_t}}{Q_1^{s_1}\cdots Q_\ell^{s_\ell}}$$

where $P_1 \cdots P_t$ are places at which the rational function η has zeros of order $\gamma_1,\cdots,\gamma_t$ and $Q_1,\cdots Q_\ell$ are places at which it has poles of order $s_1,\cdots,s_\ell$

4. At every cycle Q of Q^s of the denominator, the expansion for the rational function η has the form,

$$\eta = \frac{\beta}{t^s} + \cdots\cdots$$

where β is a constant different from zero. At other cycles this expansion has the form,

$$\eta = \alpha + \beta t^{\gamma} + \cdots$$