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SUBMANIFOLDS WITH PARALLEL MEAN CURVATURE VECTOR IN A SPHERE

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Abstract

In this paper, we study submanifolds in a unit sphere with parallel mean curvature vector, a formula of Simons' type is obtained and a corresponding pinching theorem is proved.

1. Introduction

Let M be a closed n-dimensional Riemannian manifold immersed in the unit sphere S^{n+p} of dimension $n+p$. Denote by S the square of the length of the second fundamental form and by H the mean curvature of M. When M is minimal, J. Simons [9] obtained a pinching constant $n/(2-p^{-1})$ of S and Chern, do Carmo and Kobayashi [5] proved that if $S \le n/(2-p^{-1})$ on M, then either M is totally geodesic, or the equality holds and M is either a Clifford hypersurface or a Veronese surface in S^4. Then A. M. Li and J. M. Li [7] obtained a better pinching constant $(2/3)n$ of S and proved that if $S \le (2/3)n$ on M, then M is either totally geodesic or a Veronese surface in S^4. When M has parallel mean curvature vector, Z. H. Hou [6] obtained a pinching constant $2\sqrt{n-1}$ of S for the case of $p=1$, i.e., M is a hypersurface of constant mean curvature immersed in the unit sphere, and characterized all such hypersurfaces with $S \le 2\sqrt{n-1}$. On the other hand G. Chen and X. Zou [4] discussed the case of $p>2$ and proved that if $2 \le n \le 7$ and $S \le (2/3)n$ on M, then M is totally umbilical.

In this paper, we prove the following:

THEOREM 1. *Let M be a closed n-dimensinal Riemannian manifold immersed in the unit sphere S^{n+p} of dimension $n+p$, $p \ge 2$. If the mean curvature vector of M is non-zero parallel, then*

$$\int_M (aS-n)(S-nH^2) * 1 \ge 0, \tag{1.1}$$

where $a = \max\{3/2, n/2\sqrt{n-1}\}$ and $*1$ denotes the volume element of M.

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