

HOLOMORPHIC GEODESIC TRANSFORMATIONS

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Abstract

We treat holomorphic geodesic transformations with respect to points and (holomorphic) submanifolds in an almost Hermitian manifold. We derive necessary and sufficient conditions for the existence and study how it influences the geometry of the submanifold. Furthermore, we use these transformations to characterize locally Hermitian symmetric spaces and complex space forms. Also, we determine all holomorphic geodesic transformations in such space forms.

1. Introduction

Roughly speaking, *geodesic transformations* with respect to points or submanifolds P in a Riemannian manifold M are local diffeomorphisms which transform a tubular hypersurface about P into another tubular hypersurface by moving points along normal geodesics of P and leaving the points of P fixed. These transformations are extensions of geodesic symmetries and local reflections with respect to submanifolds. They have been introduced in [5], [13] (see also [3]). In [5]–[8], we studied conformal and divergence-preserving geodesic transformations and used them to characterize real, complex and quaternionic space forms and harmonic spaces as well as special classes of submanifolds. Isoparametric hypersurfaces in real space forms or Hopf hypersurfaces with constant principal curvatures in complex space forms are typical examples.

For an almost Hermitian manifold M , *isometric*, *symplectic* and *holomorphic* geodesic symmetries and reflections with respect to submanifolds have been treated in [2], [12] (see also [14]). As has been shown in [5], [7], an isometric or symplectic geodesic transformation reduces automatically to the identity map or a local reflection. For that reason, we focus here on the study of *holomorphic* geodesic transformations. In Section 2, we begin by collecting some useful

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