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KODAI MATH. J.
20 (1997), 143—155

ON THE FREQUENCY OF ZEROS OF A FUNDAMENTAL SOLUTION SET OF COMPLEX LINEAR DIFFERENTIAL EQUATIONS

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Abstract

We give a complete characterization of those equations of the form (1.1) which possess a fundamental solution set having few zeros. Some other results are also obtained.

1. Introduction and Results

Consider a linear differential equation of the form

$$f^{(n)}+p_{n-1}(z)f^{(n-1)}+\cdots+p_0(z)f=0, \tag{1.1}$$

where $n\geqq 2$ and $p_0(z), \dots, p_{n-1}(z)$ are polynomials with $p_0(z)\not\equiv 0$. It is well known that every solution f of equation (1.1) is either a polynomial or a transcendental entire function of positive rational order. Moreover, there are at most n distinct positive rational numbers which form the set of all the possible orders of transcendental solutions of equation (1.1). This list of rational numbers can be obtained either from the Newton-Puiseux diagram ([12], [13]), or from a simple arithmetic, which was developed in [7], with the degrees of the polynomial coefficients in (1.1). The interesting reader may refer to [7] for more specific information about the possible orders of transcendental solutions of an equation of the form (1.1).

In what follows, we denote the order of growth of an entire function f by $\rho(f)$, and the exponent of convergence of its zeros by $\lambda(f)$. We assume that the coefficients $p_0(z), \dots, p_{n-1}(z)$ in equation (1.1) are not all constants.

For equation (1.1), set $d_j=\deg p_j(z)$ and

$$\gamma=1+\max_{0\leq j\leq n-1}\frac{d_j}{n-j}. \tag{1.2}$$

Then it is known [8, p. 127] that any solution f of (1.1) satisfies

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1991 Mathematics Subject Classification: Primary 34A20; Secondary 30D35.
Received December 11, 1996.