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MEROMORPHIC FUNCTIONS THAT SHARE TWO SETS

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Abstract

This paper studies the problem of uniqueness of meromorphic functions that share two sets and obtains some unicity theorems which improve some theorems given by H.X. Yi, G.D. Song and N. Li and other authors.

1. Introduction

In this paper, by meromorphic function we always mean a meromorphic function in the complex plane $\boldsymbol{C}$. We adopt the usual notations in the Nevanlinna theory of meromorphic functions as explained in [1]. Let h be a nonconstant meromorphic function and let S be a subset of distinct elements in $\hat{C}$. Define

$$E_h(S)=\bigcup_{a\in S}\{z\,|\,h(z)-a=0\},$$

where each zero of $h(z)-a=0$ with multiplicity m is repeated m times in $E_h(S)$ (see [2]). The notation $\bar{E}_h(S)$ expresses the set which contains the same points as $E_h(S)$ but without counting multiplicities (see [3]).

Throughout this paper, we assume that f and g are two nonconstant meromorphic functions, S is a subset of distinct elements in $\hat{\boldsymbol{C}}$. If $E_f(S)=E_g(S)$, we say f and g share the set S CM (counting multiplicity). If $\bar{E}_f(S)=\bar{E}_g(S)$, we say f and g share the set S IM (ignoring multiplicity). As a special case, let $S=\{a\}$, where $a\in\hat{\boldsymbol{C}}$. If $E_f(\{a\})=E_g(\{a\})$, we say f and g share the value a CM. If $\bar{E}_f(\{a\})=\bar{E}_g(\{a\})$, we say f and g share the value a IM (see [4]).

In 1976, F. Gross and C.F. Osgood proved the following theorem.

THEOREM A [5]. *Let $S_1=\{-1, 1\}$, $S_2=\{0\}$. If f and g are nonconstant entire functions of finite order such that f and g share the sets S_1 and S_2 CM, then $f\equiv\pm g$ or $f\cdot g\equiv\pm 1$.*

Next, we assume that $S_1=\{1, \omega, \dots, \omega^{n-1}\}$, $S_2=\{\infty\}$, and $S_3=\{0\}$, where $\omega=\cos(2\pi/n)+i\sin(2\pi/n)$, n is a positive integer.

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