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ON THE SCHWARZIAN DIFFERENTIAL EQUATION $\{w, z\}=R(z, w)$

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Abstract

It is showed in this note that if the Schwarzian differential equation $(*)$ $\{w, z\}=R(z, w)=P(z, w)/Q(z, w)$, where $P(z, w)$ and $Q(z, w)$ are polynomials in w with meromorphic coefficients, possesses an admissible solution $w(z)$, then $w(z)$ satisfies a first order equation of the form $(**)$ $(w')^2+B(z, w)w'+A(z, w)=0$, where $B(z, w)$ and $A(z, w)$, are polynomials in w having small coefficients with respect to $w(z)$, or by a suitable Möbius transformation $(*)$ reduces into $\{w, z\}=P(z, w)/(w+b(z))^2$ or $\{w, z\}=c(z)$. Furthermore, we study the equation $(**)$.

1. Introduction

We are concerned with the Schwarzian differential equation

$$\{w, z\}=\left(\frac{w''}{w'}\right)'-\frac{1}{2}\left(\frac{w''}{w'}\right)^2=R(z, w)=\frac{P(z, w)}{Q(z, w)}, \tag{1.1}$$

where $P(z, w)$ and $Q(z, w)$ are polynomials in w having meromorphic coefficients with $\deg_w P(z, w)=p$ and $\deg_w Q(z, w)=q$, respectively. Moreover, we assume that they are relatively prime.

We studied the Schwarzian equation $\{w, z\}^m=R(z, w)$ in [2, Theorems 1-3]. The Malmquist-Yoshida type theorem to the Schwarzian equation was obtained. Furthermore, we determined the form of the Schwarzian equation that possesses an admissible solution especially when $R(z, w)$ is independent of z. However, it might be difficult to get the similar assertion in the case when $R(z, w)$ is not independent of z. We treat the Schwarzian equation only when $m=1$, say, the equation (1.1). We also consider the first order equation

$$(w')^2+2B(z, w)w'+A(z, w)=0, \tag{1.2}$$

where $B(z, w)$ and $A(z, w)$ are polynomials in w having meromorphic coefficients. In this note, we use standard notations in the Nevanlinna theory (see e.g., [1], [5], [6]). Let $f(z)$ be a meromorphic function. Here, the word "meromorphic" means meromorphic in $|z|<\infty$. As usual, $m(r, f)$, $N(r, f)$, and $T(r, f)$ denote

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