R. WANG
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LINEAR ISOMETRIC OPERATORS ON THE $C_0^{(n)}(X)$ TYPE SPACES

RISHENG WANG

Abstract

In this paper, we try to investigate the representations of isometries, isometry groups and the space classifications of the $C_0^{(n)}(X)$ type spaces ($X\subset \boldsymbol{R}^m, m, n\geqq 1$).

§ 0. Introduction

Let $\boldsymbol{Z}_+$ be the set of non-negative integers. We make the following notations:

$$\boldsymbol{x}=(x_1, x_2, \cdots, x_m)\in \boldsymbol{R}^m \quad \boldsymbol{r}=(r_1, r_2, \cdots, r_m)\in \boldsymbol{Z}_+^m$$

$$\boldsymbol{r}!=r_1!r_2!\cdots r_m! \qquad |\boldsymbol{r}|=r_1+r_2+\cdots+r_m$$

$$f^{(\boldsymbol{r})}(\boldsymbol{x})=\frac{\partial^{r_1+r_2+\cdots+r_m}f(\boldsymbol{x})}{\partial x_1^{r_1}\partial x_2^{r_2}\cdots\partial x_m^{r_m}}.$$

If Ω is a locally compact Hausdorff space, $C_0(\Omega)$ denotes the Banach space consisting of continuous function f on Ω vanishing at infinity (i.e., $\{\omega\in\Omega: |f(\omega)|\geqq\varepsilon\}$ is compact for all $\varepsilon>0$), with the norm $\|f\|=\sup\{|f(\omega)|:\omega\in\Omega\}$. For any integers m, $n\geqq 1$, set $\Gamma=\{\boldsymbol{r}=(r_1, \cdots, r_m)\in\boldsymbol{Z}_+^m: r_1+\cdots+r_m\leqq n\}$. A subset X of $\boldsymbol{R}^m$ is called to be NIP: if for any line L parallel to one of the axes of $\boldsymbol{R}^m$ the set $L\cap X$ contains no isolated points. If X is a locally compact and NIP subset of $\boldsymbol{R}^m$, we use $C_0^{(n)}(X)$ to denote the normed space consisting of all function f on X which satisfies: $f^{(\boldsymbol{r})}\in C_0(X)$ for all $\boldsymbol{r}\in\Gamma$, with the norm $\|f\| =\sup_{\boldsymbol{x}\in X}\sum_{\boldsymbol{r}\in\Gamma}|f^{(\boldsymbol{r})}(\boldsymbol{x})|/\boldsymbol{r}!$. We set $C_0^{(0)}(X)=C_0(X)$ and use $S_{n,X}$ to denote the unit sphere of $C_0^{(n)}(X)$.

For the case $n=m=1$ and $X, Y\subseteqq\boldsymbol{R}^1$, the representations of surjective linear isometries between $C_0^{(1)}(X)$ and $C_0^{(1)}(Y)$ had been studied by Cambern and Pathak [1] (complex case only), for $m=1$, $n\geqq 1$ and $X=Y=[0, 1]$, by Pathak [2] (complex case only), and for $m=1$, $n\geqq 1$ and $X, Y\subseteqq\boldsymbol{R}^1$ with some conditions by

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