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THE KÄHLER-EINSTEIN METRICS ON A K3 SURFACE CANNOT BE ALMOST KÄHLER WITH RESPECT TO AN OPPOSITE ALMOST COMPLEX STRUCTURE

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§ 1. Introduction

It is a fundamental fact that an almost complex structure on a manifold has its preferred orientation of the manifold. Even for an almost complex manifold, i.e., an oriented manifold with an almost complex structure J, there is some additional obstruction for the manifold to admit another almost complex structure whose preferred orientation is opposite to that of J. (Such an obstruction has been obtained in dimension four [9].)

It is then interesting to know whether or not the choice of orientation of a manifold affects an almost complex structure on the manifold to have *good properties* such as integrability or parallelizability for some metric connections. Of course, such a problem is valid for a manifold which admits two kinds of almost complex structures with different preferred orientations.

The purpose of the present note is to observe such interesting phenomena concerning almost complex structures on 4-dimensional manifolds and the choice of orientation of the manifolds.

By an *opposite almost complex structure* on an oriented smooth 4-manifold X, we mean an almost complex structure on $-\bar{X}$ (the 4-manifold X with orientation reversed) [9]. If X does not admit an almost complex structure but an opposite almost complex structure, then it is preferable to treat $-\bar{X}$ rather than X since it can be recognized as an almost complex structure. The notion of opposite almost complex structures on 4-manifolds is, therefore, meaningful for 4-manifolds already carrying almost complex structure (almost complex manifolds) or 4-manifolds with orientation chosen primarily.

The condition for a 4-manifold to admit a pair of an almost complex structure and an opposite almost complex structure is equivalent to the existence of a field of oriented tangent 2-planes on the 4-manifold [9] (see also [7]).

A 4-manifold X with an opposite almost complex structure is said to be

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