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HARMONIC DIMENSION OF COVERING SURFACES, II

Dedicated to Professor Fumi-Yuki Maeda on his sixtieth birthday

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Introduction

Let F be an open Riemann surface of null boundary which has a single ideal boundary component in the sense of Kerékjártó-Stoïlow (cf. [3, p. 98]). A relatively noncompact subregion Ω of F is said to be an *end* of F if the relative boundary $\partial\Omega$ consists of finitely many analytic Jordan curves (cf. Heins [4]). We denote by $\mathcal{P}(\Omega)$ the class of all nonnegative harmonic functions on Ω with vanishing values on $\partial\Omega$. The *harmonic dimension* of Ω, $\dim \mathcal{P}(\Omega)$ in notation, is defined as the minimum number of elements of $\mathcal{P}(\Omega)$ generating $\mathcal{P}(\Omega)$ provided that such a finite set exists, otherwise as ∞. It is well-known that $\dim \mathcal{P}(\Omega)$ dose not depend on a choice of end of F: $\dim \mathcal{P}(\Omega)=\dim \mathcal{P}(\Omega')$ for any pair (Ω, Ω') of ends of F (cf. [4]). In terms of the Martin compactification $\dim \mathcal{P}(\Omega)$ coincides with the number of minimal points over the ideal boundary (cf. Constantinesc and Cornea [3]).

In this note we especially consider ends W which are subregion of p-sheeted unlimited covering surfaces of $\{0<|z|\leqq\infty\}$. For these W it is known that $1\leqq\dim \mathcal{P}(W)\leqq p$ (cf. [4]). Consider two positive sequences $\{a_n\}$ and $\{b_n\}$ satisfying $b_{n+1}<a_n<b_n<1$ and $\lim_{n\to\infty} a_n=0$. Set $G=\{0<|z|<1\}-I$ where $I=\bigcup_{n=1}^{\infty} I_n$ and $I_n=[a_n, b_n]$. We take p (>1) copies $G_1, \cdots, G_p$ of G. Joining the upper edge of I_n on G_j and the lower edge of I_n on G_{j+1} ($j \bmod p$) for every n, we obtain a p-sheeted covering surface $W=W_p^I$ of $\{0<|z|<1\}$ which is naturally considered as an end of a p-sheeted covering surface of $\{0<|z|\leqq\infty\}$. In the previous paper [6] we proved the following.

THEOREM A ([6, Theorem]). *Suppose that* $p=2^m$ $(m\in \boldsymbol{N})$. *Then*
(i) $\dim \mathcal{P}(W)=p$ *if and only if* I *is thin at* $z=0$;
(ii) $\dim \mathcal{P}(W)=1$ *if and only if* I *is not thin at* $z=0$.

The purpose of this note is to show that, in a bit more general setting for I, Theorem A is valid for every p (>1) (cf. § 1). Consequently we have the following.

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