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SINGULAR VARIATION OF DOMAINS AND CONTINUITY PROPERTY OF EIGENFUNCTION FOR SOME SEMI-LINEAR ELLIPTIC EQUATIONS

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1. Introduction

Let M be a bounded domain in $\boldsymbol{R}^3$ with smooth boundary ∂M. Let w be a fixed point in M. By $B(\varepsilon; w)$ we denote the ball of center w with radius ε. We remove $\overline{B(\varepsilon; w)}$ from M and we put $M_\varepsilon = M \backslash \overline{B(\varepsilon; w)}$. We write $B(\varepsilon; w) = B_\varepsilon$.

Fix $k \geqq 0$ and $p \in (1, 5)$. We put

$$\lambda(\varepsilon) = \inf_{X_\varepsilon} \left(\int_{M_\varepsilon} |\nabla u|^2 dx + k \int_{\partial M_\varepsilon} u^2 d\sigma \right), \tag{1.1$_\varepsilon$}$$

where

$$X_\varepsilon = \{ u \in H^1(M_\varepsilon) : \|u\|_{L^{p+1}(M_\varepsilon)} = 1,\ u = 0 \text{ on } \partial M,\ u \geqq 0 \text{ in } M_\varepsilon \}.$$

Then, we know that there exists at least one solution u_ε which attains $(1.1)_\varepsilon$. It satisfies

$$\begin{aligned} &-\Delta u_\varepsilon = \lambda(\varepsilon) u_\varepsilon^p \quad \text{in} \quad M_\varepsilon \\ &\frac{\partial u_\varepsilon}{\partial \nu_x} + k u_\varepsilon = 0 \quad \text{on} \quad \partial B_\varepsilon \\ &u_\varepsilon = 0 \quad \text{on} \quad \partial M. \end{aligned} \tag{1.2}$$

Here $\partial/\partial \nu_x$ denotes the derivative along the exterior normal direction.

One of the main results of this paper is the following.

THEOREM 1. *Fix $p \in (1, 5)$. Then, there exists a constant C independent of ε such that*

$$\sup_{u_\varepsilon \in S_\varepsilon} \sup_{x \in M_\varepsilon} |u_\varepsilon(x)| \leqq C < +\infty,$$

where S_ε is the set of positive solutions of (1.2) *which minimize* $(1.1)_\varepsilon$.

Next we treat the asymptotic behaviours of $\lambda(\varepsilon)$ and positive solutions u_ε

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