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KODAI MATH. J.
17 (1994), 4—14

DECOMPOSITION NUMBERS FOR SPIN CHARACTERS OF EXCEPTIONAL WEYL GROUPS OF TYPE E_n

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Introduction

It is well known that the groups, $W(E_6)\cong S_4(3).2\cong U_4(2).2$; $W(E_7)\cong C_2\times B_3(2)\cong C_2\times D_7(2)$, where C_2 is the cyclic group of order 2 and $W(E_8)\cong 2.\ O_8^+(2).2\cong 2.\ D_4(2).\ 2$ (see, [CCN]),

In this paper, which is a continuation of [Sa], the decomposition matrices for the spin characters of the exceptional Weyl groups of type E_n ($n=6, 7, 8$), for the prime numbers $p=5, 7$ are determined. In all cases except $W(E_8)$ and $p=5$, the relevent prime number divides, the order of the group to the first power only.

First we use the central characters to split the ordinary characters into p-blocks (For general information about p-blocks, see [CR]). Let B be a p-block of a group G and let $\Psi_1, \cdots, \Psi_s$ and $\phi_1, \cdots, \phi_r$, respectively, denote the ordinary and p-modular irreducible characters of G in B. The restriction of each Ψ_i to p-regular classes of G, denoted by $\bar{\Psi}_i$ is a p-modular character of G and $\bar{\Psi}_i=\sum_{j=1}^{r} d_{ij}\phi_j$ $(1\leqq i\leqq s)$ where d_{ij}'s are non-negative ingers, called the decomposition numbers of B for the prime p. The $(s\times r)$ matrix $D_P^B(G)=(d_{ij})$ is called the decomposition matrix of B for the prime p. Furthermore, a principal character $\sum_{i=1}^{s} c_i\Psi_i$ will be identified with the column of integers $c=(c_i)$ and it may be indecomposable or sum of principal indecomposable characters in B.

One way to constructs $D_P^B(G)$ is to find the principal indecomposable characters in B. This method is described by James and Kerber [JK] and is as follows; suppose that the matrix

$$R_P^B(G)=(c^{(0)}, c^{(1)}, \cdots, c^{(n_1)}, \cdots, c_{r-1}^{(0)}, c_{r-1}^{(1)}, \cdots, c_{r-1}^{(n_{r-1})}, c_r)$$

of principal characters has been found such that, with a suitable arrangement of the ordinary irreducible characters, for $0\leqq k_i\leqq n_i$ and $i=1, \cdots, (r-1)$, each of the matrices

$$R_P^B(G)=(c_1^{(k_1)}, \cdots, c_{r-1}^{(k_{r-1})}, c_r)$$

has the triangle of zeros above the main diagonal and 1's on the main diagonal.

Received October 30, 1992; revised August 23, 1993.