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ON AN ULTRAHYPERELLIPTIC SURFACE WITH PICARD CONSTANT THREE

Dedicated to Professor Nobuyuki Suita on his 60th birthday

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§ 1. Introduction.

Let R be an open Riemann surface. Let $M(R)$ be the family of non-constant meromorphic functions on R. Let $P(f)$ be the number of values which are not taken by $f\in M(R)$. The Picard constant $P(R)$ of R is defined by

$$P(R)=\sup\{P(f)\,;\ f\in M(R)\}$$

Then we have $P(R)\geqq 2$. The significance of the Picard constant is in the following fact: *If $P(R)<P(S)$, then there is no non-constant analytic mapping of R into S* (Ozawa [5]).

Let R be the ultrahyperelliptic surface defined by

$$y^2=G(z), \tag{1.1}$$

where G is an entire function having an infinite number of simple zeros and no other zeros. For the class of this surfaces we have $P(R)\leqq 4$ from the value distribution theory of two-valued algebroid functions.

We now consider a characterization of ultrahyperelliptic surfaces in terms of the Picard constant. We first have

THEOREM A (Ozawa [6]). *$P(R)=4$, if and only if there is a non-constant entire function H $(H(0)=0)$, an entire function F and constants γ and δ such that G in* (1.1) *satisfies*

$$F(z)^2G(z)=(e^{H(z)}-\gamma)(e^{H(z)}-\delta), \qquad \gamma\delta(\gamma-\delta)\neq 0\,. \tag{1.2}$$

When $P(R)=3$ we have

THEOREM B (Hiromi-Ozawa [1]). *If $P(R)=3$, then there are two non-constant entire functions H and L $(H(0)=L(0)=0)$, an entire function F and non-zero constants β_1 and β_2 such that G in* (1.1) *satisfies*

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