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ON THE ZEROS OF A HOMOGENEOUS DIFFERENTIAL POLYNOMIAL

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1. Introduction

Concerning a question of W. K. Hayman [2] (see also [3: Problem 1.18], [4]) E. Mues [5] discussed entire functions g in which the homogeneous differential polynomial $g''g-ag'^2$ has no zeros. He proved that

$$g(z)=\exp(\alpha z+\beta), \qquad \alpha(\neq 0),\ \beta\in \boldsymbol{C}$$

are the only transcendental entire functions with this property if $a\neq 1$. Giving the following two counter-examples he also showed that the case where $a=1$ is indeed exceptional:

(a) $$g(z)=\sin z$$

and

(b) $$g(z)=\exp\{Q(z)-h(z)\},$$

provided that $h(z)$ is an arbitrary entire function and $Q(z)$ is an entire function defined by

$$Q(z)=\int_{z_0}^{z}\int_{\zeta_0}^{\zeta}\{h''(t)+\exp(2h(t))\}\,dt\,d\zeta.$$

In this paper we discuss the corresponding results to meromorphic functions g when there somewhat exist both the poles of g and the zeros of the homogeneous differential polynomial $g''g-ag'^2$. By a meromorphic function we mean a function meromorphic in the complex plane $\boldsymbol{C}$. We follow the notation and terminology of [2] and [4]. We shall explain the special symbols whenever we introduce them. In particular, if f is a meromorphic function, we shall denote by $S(r, f)$ any quantity

(1.0) $$S(r, f)=o\{T(r, f)\}$$

as $r\to\infty$, possibly outside a set of r of finite linear measure. For the sake of

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