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KO-HOMOLOGIES OF A FEW CELLS COMPLEXES

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0. Introduction.

Let KO and KU be the real and the complex K-spectrum respectively. For any CW-spectra X and Y we say that X is *quasi KO_*-equivalent* to Y if there exists a map $h: Y \to KO \wedge X$ such that the composite map $(\mu \wedge 1)(1 \wedge h)$: $KO \wedge Y \to KO \wedge X$ is an equivalence where μ: $KO \wedge KO \to KO$ denotes the multiplication of KO (see [4] or [3]). Such a map h is called to be a quasi KO_*-equivalence. If X is quasi KO_*-equivalent to Y, then KO_*X is isomorphic to KO_*Y as a KO_*-module and in addition KU_*X is isomorphic to KU_*Y as an abelian group with involution where the conjugation ψ_C^{-1} behaves as an involution. Assume that CW-spectra X and Z have the same quasi KO_*-types as CW-spectra Y and W respectively. For any maps f: $Z \to X$ and g: $W \to Y$ we say that f is *quasi KO_*-equivalent* to g if there exist KO_*-equivalences $h: Y \to KO \wedge X$ and $k: W \to KO \wedge Z$ such that the equality $hg = (1 \wedge f)k: W \to KO \wedge X$ holds. In this case their cofibers $C(f)$ and $C(g)$ have the same quasi KO_*-type.

A CW-spectrum X is said to be *stably* quasi KO_*-equivalent to a CW-spectrum Y if X is quasi KO_*-equivalent to the i-fold suspended spectrum $\Sigma^i Y$ for some i. In this note we shall be interested in the stable quasi KO_*-types of complexes with a few cells. Each complex with 2-cells is stably quasi KO_*-equivalent to one of the following spectra $\Sigma^0 \vee \Sigma^i$ $(0 \leqq i \leqq 7)$, SZ/t $(t \geqq 1)$, $P = C(\eta)$ and $Q = C(\eta^2)$ where SZ/t denotes the Moore spectrum of type Z/t and η: $\Sigma^1 \to \Sigma^0$ is the stable Hopf map of order 2. Our purpose of this paper is to determine the stable quasi KO_*-types of any complexes with 3- or 4-cells (Theorems 5.3 and 5.4). In [4] and [5] we introduced some 3-cells spectra X_m and X'_m constructed as the cofibers of certain maps f: $\Sigma^i \to SZ/2^m$ and f': $\Sigma^i SZ/2^m \to \Sigma^0$ and some 4-cells spectra XY_m, $X'Y'_m$ and $Y'X_m$ obtained as the cofibers of their mixed maps. In § 1 and § 4 we study the quasi KO_*-types of their cofibers C(g) for any maps g: $S_i \to \Delta X$ realizing elements of $KO_i X$ when $X = SZ/2^m$, P, Q, X_m or X'_m. In § 2 we introduce some 4-cells spectra $X_{m,n}$ constructed as the cofibers of certain maps $f: \Sigma^i SZ/2^n \to SZ/2^m$, and then study the quasi KO_*-types of their cofibers $C(g)$ for any maps $g: \Sigma^i SZ_n \to \Delta X$ realizing elements of $[\Sigma^i SZ/2^n, KO \wedge X]$ when $X = SZ/2^m$, P or Q. In § 3 we introduce some new small spectra $XV_{m,n}$, $VX_{m,n}$ and $X'X_{n,m}$ needed in § 4. In § 5 we prove Theorems 5.3 and 5.4 by using results obtaind in §§ 1-4.

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