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SINGULAR VARIATION OF DOMAIN AND SPECTRA OF THE LAPLACIAN WITH SMALL ROBIN CONDITIONAL BOUNDARY II

Dedicated to Professor Takeshi Watanabe on his 60th birthday

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1. Introduction.

This paper is a continuation of previous paper [6].

Let Ω be a bounded domain in $\boldsymbol{R}^2$ with smooth boundary $\partial\Omega$. Let $\tilde{w}$ be a fixed point point in Ω. Let $B(\varepsilon, \tilde{w})$ be the disk of radius ε with the center $\tilde{w}$. We put $\Omega_\varepsilon=\Omega\setminus\overline{B(\varepsilon, \tilde{w})}$. Consider the following eigenvalue problem

$$(1.1)\qquad \begin{gathered} -\Delta u(x)=\lambda u(x) \qquad x\in\Omega_\varepsilon \\ u(x)=0 \qquad x\in\partial\Omega \\ u(x)+k\varepsilon^\sigma \frac{\partial u}{\partial\nu_x}(x)=0 \qquad x\in\partial B(\varepsilon, \tilde{w}). \end{gathered}$$

Here k denotes the positive constant. And σ is a real number. Here $\partial/\partial\nu_x$ denotes the derivative along the exterior normal direction with respect to Ω_ε.

Let $\mu_j(\varepsilon)>0$ be the j-th eigenvalue of (1.1). Let μ_j be the j-th eigenvalue of the problem

$$(1.2)\qquad \begin{gathered} -\Delta u(x)=\lambda u(x) \qquad x\in\Omega \\ u(x)=0 \qquad x\in\partial\Omega . \end{gathered}$$

Let $G(x, y)$ be the Green function of the Laplacian in Ω associated with the boundary condition (1.2).

Main aim of this paper is to show the following Theorems. Let $\varphi_j(x)$ be the L^2-normalized eigenfunction associated with μ_j. We have the following.

THEOREM 1. *Assume that μ_j is a simple eigenvalue. Then,*

$$\mu_j(\varepsilon)=\mu_j-2\pi\varphi_j(\tilde{w})^2/(\log \varepsilon)+O(|\log \varepsilon|^{-2}),$$

for $\sigma\geqq 1$.

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