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COMPLETE SPACE-LIKE HYPERSURFACES WITH CONSTANT MEAN CURVATURE IN A LORENTZ SPACE FORM OF DIMENSION 4

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Abstract

On complete space-like hypersurfaces with constant mean curvature in a Lorentz space form of dimension 4, we study the case that the scalar curvature is constant and that the Ricci curvature is bounded from above.

1. Introduction.

Let $\boldsymbol{R}_1^{n+1}$ be an $(n+1)$-dimensional Minkowski space and $\boldsymbol{S}_1^{n+1}(c)$ (resp. $\boldsymbol{H}_1^{n+1}(c)$) be an $(n+1)$-dimensional de Sitter space (resp. anti-de Sitter space) of constant curvature c. Considered collectively, a Lorentz manifold of these kinds is called a *Lorentz space form* of constant curvature c, which is denoted by $M_1^{n+1}(c)$.

Since Calabi [4] and S.Y. Cheng and Yau [7] proved the Bernstein type theorem in $\boldsymbol{R}_1^{n+1}$, complete space-like hypersurfaces with constant mean curvature in a Lorentz space form $M_1^{n+1}(c)$ have been studying by many mathematicians. On the other hand, space-like hypersurfaces with constant mean curvature in spacetimes get interested in relativity theory.

It is well known that totally umbilical hypersurfaces $M^n(c')$ $(c'<c)$ and hypersurfaces in the form of $\boldsymbol{H}^k(c_1)\times M^{n-k}(c_2)$ $[k=1, \cdots, n-1, c_1<0, c(c_1+c_2)=c_1c_2]$ are standard models of complete space-like hypersurfaces with constant mean curvature in $M_1^{n+1}(c)$. Here $M^n(c)$ means an n-dimensional space form with constant curvature c, that is, a Riemannian sphere $\boldsymbol{S}^n(c)$, a hyperbolic space $\boldsymbol{H}^n(c)$ or a Euclidean space $\boldsymbol{R}^n$.

Let M be a complete space-like hypersurface with constant mean curvature h/n in $M_1^{n+1}(c)$. In a de Sitter space $\boldsymbol{S}_1^{n+1}(c)$, M is nothing but totally umbilical if $n=2$ and $h^2\leqq 4c$ or if $n>2$ and $h^2<4(n-1)c$ (cf. Akutagawa [3], Ramanathan [12] or Cheng [5]).

In the other case, there are many examples in $M_1^{n+1}(c)$ which are not standard models (cf. Treibergs [13], Ishihara and Hara [8], Akutagawa [3] and others). But we have known some characterizations of standard models with respect to

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