T. OSAWA
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THE HADAMARD VARIATIONAL FORMULA FOR THE GROUND STATE VALUE OF $-\Delta u=\lambda|u|^{p-1}u$

BY TATSUZO OSAWA

1. Introduction. This article is divided into three parts. In every part we study the Hadamard variational formula for the (non-trivial) ground state value of the semi-linear equation $-\Delta u=\lambda|u|^{p-1}u$.

Let Ω be a bounded domain in R^N $(N\geqq 2)$ with smooth boundary $\partial\Omega$. Let ρ be a smooth function on $\partial\Omega$. We denote $\nu(x)$ as the exterior unit normal vector at $x\in\partial\Omega$. If ε is small enough, we have a new domain Ω_ε bounded by

$$\partial\Omega_\varepsilon=\{x+\varepsilon\rho(x)\nu(x);\ x\in\partial\Omega\}.$$

Let p be a fixed number satisfying $1<p<\infty$ for $N=2$, $1<p<(N+2)/(N-2)$ for $N\geqq 3$.

We consider the minimizing problem

$$\lambda_\varepsilon=\inf_{X_\varepsilon}\int_{\Omega_\varepsilon}|\nabla\varphi|^2dx, \tag{1.1}$$

where

$$X_\varepsilon=\{\varphi\in H_0^1(\Omega_\varepsilon),\ \varphi\geqq 0,\ \|\varphi\|_{L^{p+1}(\Omega_\varepsilon)}=1\}.$$

For the sake of simplicity we write $\|\ \|_{L^{p+1}(\Omega_\varepsilon)}$ as $\|\ \|_{p+1,\varepsilon}$. It is well known that there exists at least one solution $u_\varepsilon\in C^{3,\alpha}(\bar{\Omega}_\varepsilon)$ satisfying $\|u_\varepsilon\|_{p+1,\varepsilon}=1$, and

$$\begin{aligned}-\Delta u_\varepsilon(x)&=\lambda_\varepsilon u_\varepsilon^p(x) && x\in\Omega_\varepsilon\\ u_\varepsilon(x)&=0 && x\in\partial\Omega_\varepsilon,\end{aligned}$$

and $u_\varepsilon>0$ in Ω_ε.

The author calls λ_ε as the Dirichlet ground state value on Ω_ε and u_ε as the Dirichlet ground state solution.

In this note we would like to consider ε-dependence of λ_ε, u_ε. One of the main result of this paper is the following: Here $\lambda_0=\lambda$, $u_0=u$.

THEOREM 1. *Assume that the number of positive solution u which minimize $(1.1)_0$ is unique. Assume that* $\mathrm{Ker}(\Delta+\lambda p u^{p-1})=\{0\}$. *Then, we have the following limit.*

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