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ON THE FREQUENCY OF COMPLEX ZEROS OF SOLUTIONS OF CERTAIN DIFFERENTIAL EQUATIONS

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Abstract.

In this paper, we investigate the frequency of zeros of solutions of linear differential equations of the form $w^{(k)}+\sum_{j=1}^{k-1} Q_j w^{(j)}+(Q_0+Re^P)w=0$, where $k\geqq 2$, and where $Q_0, \cdots, Q_{k-1}$, R and P are arbitrary polynomials with $R\not\equiv 0$ and P non-constant. All solutions $f\not\equiv 0$ of such an equation are entire functions of infinite order of growth, but there are examples of such equations which can possess a solution whose zero-sequence has a finite exponent of convergence. In this paper, we show that unless a special relation exists between the polynomials $Q_0, \cdots, Q_{k-1}$, and P, all solutions of such an equation have an infinite exponent of convergence for their zero-sequences. This result extends earlier results for the equation, $w^{(k)}+(Q_0+Re^P)w=0$.

1. Introduction. Several recent papers (e.g. [7], [8], [9], [10], [11], [15]) have dealt with the investigation of the frequency of zeros of solutions of equations of the form,

$$w^{(k)}+(Re^P+Q)w=0, \tag{1.1}$$

where $k\geqq 2$, and where R, P, and Q are polynomials with $R\not\equiv 0$ and P non-constant. It was shown in [7; §5(b), p. 356] that for any polynomial $P(z)$ of degree $r\geqq 1$, there exists a polynomial $Q(z)$ of degree $2r-2$ such that the second-order equation,

$$w''+(e^P+Q)w=0, \tag{1.2}$$

possesses two linearly independent solutions each having no zeros. This result led to an investigation in [8] of the more general equation (1.1) of arbitrary order $k\geqq 2$, and it was shown in [8] that if the degree of Q is less than $kr-k$,

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