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SOME RESULTS ON THE COMPLEX OSCILLATION THEORY OF SECOND ORDER LINEAR DIFFERENTIAL EQUATIONS

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1. Introduction

We consider the second order linear differential equation

$$f''+Af=0, \tag{1}$$

where A is an entire function. For an entire function f, let $\rho(f)$ be its order, $\mu(f)$ its lower order and $\lambda(f)$ the exponent of convergence of its zeros. In addition, we assume that the reader is familiar with the standard notations of Nevanlinna theory (see [3]).

When A is a polynomial of degree $n\geqq 1$, S. Bank and I. Laine obtained the following ([1]).

THEOREM A. *Let A be a polynomial of degree $n\geqq 1$. If $f\not\equiv 0$ is a solution of* (1), *then*

$$\rho(f)=(n+2)/2, \tag{2}$$

and if f_1, f_2 are two linearly independent solutions of (1), *then*

$$\max(\lambda(f_1), \lambda(f_2))=(n+2)/2. \tag{3}$$

If A is transcendental, we apply the lemma on the logarithmic derivative in Nevanlinna theory to (1) and can deduce that any solution $f\not\equiv 0$ of (1) satisfies

$$\rho(f)=\infty. \tag{4}$$

We may hope that

$$\max(\lambda(f_1), \lambda(f_2))=\infty, \tag{5}$$

where f_1 and f_2 are linearly independent solutions of (1). However, examples in [1] show that this is not the case. Specifically, for $\rho(A)$ a positive integer or infinity, there exist A and independent solutions f_1, f_2 of (1) such that

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