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ON Z_p-EXTENSIONS OF REAL ABELIAN FIELDS

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In [1], Fukuda and Komatsu gave a sufficient condition for the λ invariants of real quadratic fields to vanish. In this note, we shall show that the results obtained in [1] can be extended in a real abelian case.

Let k be a finite real abelian extension of Q and p an odd prime number which splits completely in k. Let $\mathcal{P}_1\mathcal{P}_2\cdots\mathcal{P}_r$ be the prime factorization of (p) in k, where $r=[k:Q]$. A Z_p-extension

$$k=k_0\subset k_1\subset\cdots\subset k_n\subset\cdots\subset K$$

is the cyclotomic Z_p-extension, since k is a totally real field and Leopoldt's conjecture is valid for k. Let C_n be the ideal class group of k_n, A_n the p-primary part of C_n, σ a topological generator of $\mathrm{Gal}(K/k)$ and we denote also by σ the restriction of σ to k_n. Let

$$C'_n=\{c\in C_n\,;\,c^\sigma=c\} \quad\text{and}\quad B_n=C'_n\cap A_n\,.$$

We have the following result by Greenberg [2], since every $\mathcal{P}_i$ is totally ramified in K.

LEMMA 1. *There exists an integer n_0 such that* $|B_0|\leqq|B_1|\leqq\cdots\leqq|B_{n_0}|=|B_{n_0+1}|=\cdots=|B_n|=\cdots$.

Let E be the unit group of k. Then

$$E=\{1,\,-1\}\oplus E'\,,$$

where E' is a free abelian group of rank $r-1$. We denote by $N_{m,n}$ the norm mapping from k_m to k_n $(m\geqq n)$. Let $H=N_{n,0}(k_n)\cap E$, then there exists a base $\{\eta_1,\eta_2,\cdots,\eta_{r-1}\}$ of E' and positive integers $c_1, c_2,\cdots, c_{r-1}$ such that

$$H=\{1,\,-1\}\oplus[\eta_1^{c_1},\,\eta_2^{c_2},\,\cdots,\,\eta_{r-1}^{c_{r-1}}] \tag{1}$$

and $c_i|c_{i+1}$ for $i=1,\cdots,r-2$. Since $[k_n:k]=p^n$,

$$H\supset\{1,\,-1\}\oplus[\eta_1^{p^n},\,\eta_2^{p^n},\,\cdots,\,\eta_{r-1}^{p^n}]\,.$$

Then we have

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