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KODAI MATH. J.
12 (1989), 346—391

A CERTAIN PROPERTY OF GEODESICS OF THE FAMILY OF RIEMANNIAN MANIFOLDS $O_n^2(X)$

Dedicated to Professor Shiing-Shen Chern on his 77th birthday

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§ 0. **Introduction.** This is a continuation of Part (IX) ([22]) with the same title by the present author which proved the following conjecture is true for $2.4 \leqq n \leqq 4.5$ and exactly the final one of the series (I)-(X). He will show that this conjecture is also true for $2 \leqq n \leqq 2.4$ in the present paper by developing a new method which is applicable for all values of $n \geqq 2.4$. As stated at the end of the previous one, the principle used until now could not hold for $2 \leqq n \leqq 2.38$. We shall also use the same notation in the previous papers (I)-(IX). Any geodesic of the 2-dimensional Riemannian manifolds O_n^2 defined on the unit disk $u^2+v^2<1$ of the uv-plane by the metric:

$$ds^2=(1-u^2-v^2)^{n-2}\{(1-v^2)du^2+2uvdudv+(1-u^2)dv^2\}$$

has the support function $x(t)$ which is a solution of the non linear differential equation of order 2([6]):

$$\text{(E)} \qquad nx(1-x^2)\frac{d^2x}{dt^2}+\left(\frac{dx}{dt}\right)^2+(1-x^2)(nx^2-1)=0$$

When the parameter $n>1$, any non-constant solution $x(t)$ of (E) such that $x^2+x'^2<1$ is periodic and its period T is given by the improper integral ([10]):

$$\text{(0.1)} \qquad T=\sqrt{nc}\int_{x_0}^{x_1}\frac{dx}{x\sqrt{(n-x)\{x(n-x)^{n-1}-c\}}},$$

where $x_0=n\{\min x(t)\}^2$, $x_1=n\{\max x(t)\}^2$, $0<x_0<1<x_1<n$ and $c=x_0(n-x_0)^{n-1}=x_1(n-x_1)^{n-1}$.

CONJECTURE C. *The period T as a function of $\tau=(x_1-1)/(n-1)$ and n is monotone decreasing with respect to $n(>2)$ for any fixed $\tau(0<\tau<1)$.*

§ 1. Preliminaries.

Setting $T=\Omega(\tau, n)$, we have the formulas

Received November 11, 1988.