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KODAI MATH. J.
12 (1989), 98–108

ON THE EXISTENCE OF LIMIT CYCLES OF THE EQUATION $x'=h(y)-F(x)$, $y'=-g(x)$ *

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1. Introduction.

Owing to their theoretical and practical importance, the Liénard equations have attracted much attention in recent years, in particular, for theory of periodic solutions, see [1-7].

In this paper, we consider the existence of limit cycles of the system

$$\begin{cases} x'=h(y)-F(x) \\ y'=-g(x), \end{cases} \tag{1}$$

which is little more general than the Liénard equation. We assume that F, G, $h: \boldsymbol{R}\to\boldsymbol{R}$ are continuous functions and satisfy the property of uniqueness for the solutions to the Cauchy problems associated to the system (1), and $xg(x)>0$ for every $x\neq 0$, $yh(y)>0$ for every $y\neq 0$. Without loss of generality, we also assume $F(0)=0$. We obtained some new results. The theorems of this paper generalized some results in [5] and [7].

Let Y^+, Y^-, C^+, C^- denote the sets $\{(x, y): y\geqq 0, x=0\}$, $\{(x, y): y\leqq 0, x=0\}$, $\{(x, y): h(y)=F(x), x>0\}$ and $\{(x, y): h(y)=F(x), x<0\}$, respectively.

2. Technical Preliminaries.

LEMMA 1. *If we assume*

(i) $$\overline{\lim_{y\to+\infty}}\, h(y)=+\infty, \quad \textit{and} \quad \underline{\lim_{y\to-\infty}}\, h(y)=-\infty,$$

then the sufficient and necessary condition that there exists a point $N\in Y^-$ such that the negative half-trajectory L_N^- passing through point N does not intersect C^+ is

(ii)$_a$ *there exists a continuously differentiable function $k_1(x)$ defined on $(0, \infty)$ with positive derivative such that*

$$F(x)\geqq h(k_1(x))+\frac{g(x)}{k_1'(x)} \qquad \textit{for} \quad x>0.$$

* Supported by the National Natural Science Foundation of China
Received December 18, 1987, Revised August 10, 1988