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KODAI MATH. J.
10 (1987), 285—291

ON UNIQUE FACTORIZABILITY OF COMPOSITE ENTIRE FUNCTIONS

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1. Introduction and main results.

There is a fundamental problem on the factorization theory of entire and meromorphic functions which is so-called "unique factorizability" of composite functions. So far only few results in this topic are known, which are mostly concerned with the unique factorizability theorems for several special types of entire functions (See, e.g., [3], [4]). For example, Ozawa [3] proved the following result.

THEOREM A. *Let $g_0(z)$ be a prime transcendental entire function of finite order, which has infinitely many zeros. Assume that almost all zeros of $g_0(z)$ lie in $ReZ \geqq x$ for every x. Then $g_0(z)^2$ is uniquely factorizable into two primes.*

We shall not state here some basic notions in the factorization theory such as prime, E-prime, pseudo-prime, left or right factor, two factorizations of a function being equivalent, etc. One may find the definitions of these notions in references of this paper. However, it seems to be necessary to give some definitions for convenience.

DEFINITION 1. Let f_0 be a non-linear meromorphic function, g_0 a non-linear entire function, both are prime. The composite function $F=f_0(g_0)$ is called uniquely factorizable if every non-trivial factorization of the form $F=f(g)$ is equivalent to $f_0(g_0)$, where f is meromorphic and g entire (g may be meromorphic when f is rational).

DEFINITION 2. Let f_0 and g_0 be two non-linear prime entire functions. The composite function $F=f_0(g_0)$ is called E-uniquely factorizable if every non-trivial factorization of the form $F=f(g)$ with entire functions f and g is equivalent to $f_0(g_0)$.

Our theorem 1 below gives a criterion that under what condition an E-uniquely factorizable entire function is uniquely factorizable. It is interesting that the criterion is quite similar to one obtained by Gross [2] concerning the connection between E-prime and prime of an entire function.

Received December 16, 1986.