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COEFFICIENT ESTIMATES FOR THE CLASS Σ

Dedicated to Professor Y. Kusunoki on his sixtieth birthday

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1. Introduction.

Let Σ denote the class of functions $f(z)$ univalent in $|z|>1$, regular apart from a simple pole at the point at infinity and having the expansion

$$f(z)=z+\sum_{n=1}^{\infty} b_n z^{-n}$$

around there. Let us introduce quantities A_n, B_n by

$$A_n=\inf\{t:\ \Re(tb_1+b_n)\leqq t,\ {}^{\forall}f\in\Sigma\},$$

$$B_n=\inf\{t:\ \Re(tb_1-b_n)\leqq t,\ {}^{\forall}f\in\Sigma\},$$

respectively. It is evident that $A_{2n}=B_{2n}$. Kirwan made a conjecture that $B_n\leqq n$ seems to be true [3]. $B_2\leqq 2$ and $B_3\leqq 3$ were due to Garabedian and Schiffer [1] and Kirwan and Schober [2] proved $B_2=2$ and $B_3=3$.

In this paper we shall prove the following

THEOREM. $A_3\leqq 2$, $A_5\leqq(27+8\sqrt{3})/12$, $A_7\leqq 5.5$, $A_9<8$, $A_{11}<10$.

$A_n\leqq n-1$ for any odd $n\geqq 3$ seems to be true. Anyway it seems to be very difficult to decide A_n exactly as well as B_n. Our method of proof depends upon the Grunsky inequality. So to explain its related notions and relations is in order here.

Let $f(z)\in\Sigma$ and let $F_m(z)$ be the mth Faber polynomial of $f(z)$, which is defined by

$$F_m(f(z))=z^m+\sum_{n=1}^{\infty} a_{mn} z^{-n}.$$

Then Grunsky's inequality has the following form

$$\left|\sum_{m,n=1}^{N} n a_{mn} x_m x_n\right|\leqq \sum_{n=1}^{N} n|x_n|^2$$

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