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ON A FREE BOUNDARY PROBLEM OF PLASMA EQUILIBRIA
—ASYMPTOTIC BEHAVIOR AND SYMMETRIC PROPERTY OF A SOLUTION—

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§ 1. Introduction.

A simple model of a confused plasma in tokomak machine can be described by the following system:

$$
\text{(E)} \qquad \begin{cases} -\Delta u=\lambda g(x, u) \quad \text{in} \quad \Omega_p=\{x\in\Omega \mid u(x)>0\}, & (1.1) \\ -\Delta u=0 \quad \text{in} \quad \Omega\setminus\Omega_p, & (1.2) \\ u|_{\partial\Omega}=\text{unknown constant}, & (1.3) \\ -\int_{\partial\Omega}\frac{\partial u}{\partial\nu}ds=I \text{ (given positive constant)}, & (1.4) \end{cases}
$$

where Ω is a bounded domain in $\boldsymbol{R}^n$ with a smooth boundary and λ is a given positive parameter. Ω_p is called a plasma domain and $\Omega\setminus\Omega_p$ is called a vacuum domain. We consider the free boundary problem of the following type:

(P) *Find*: $u\in H^2(\Omega)$ and $\Omega_p\subset\Omega$ s.t. u and Ω_p satisfy (E).

We call $\partial\Omega_p$ a free boundary.

We consider this problem under the assumptions:

$$
\text{(A1)} \qquad \begin{cases} g(x, s)=0 \quad \text{if} \quad s\leqq 0, & (1.5) \\ g(x, s)>0 \quad \text{if} \quad s>0, & (1.6) \\ g(x, s) \text{ is continuous in } \Omega\times R, \\ \lim_{s\to\infty}\frac{g(x, s)}{s^p}=0 \quad \text{uniformly in } \bar{\Omega}, \end{cases}
$$

where $p=n/(n-2)$ (if $n>2$) and $p={}^{\exists}p_0>1$ (if $n=2$). By using (1.5) and (1.6), we can rewrite (1.1) and (1.2) as follows.

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