M. OKA
KODAI MATH. J.
6 (1983), 419—433

ON THE BOUNDARY OBSTRUCTIONS TO THE JACOBIAN PROBLEM

BY MUTSUO OKA

§1. Introduction

Let K be an algebraically closed field of characteristic zero and let $f(X, Y)$ and $g(X, Y)$ be polynomials with K-coefficients which satisfy the Jacobian condition:

$$J(f, g)=f_X g_Y - f_Y g_X = 1 \tag{1.1}$$

where f_X, f_Y etc. are respective partial derivatives. The so-called Jacobian conjecture is the following.

(J.C. I) "If (1.1) is satisfied, X and Y are polynomials of f and g".

Typical examples are given by elementary transformations which are defined by finite compositions of the following transformations.

(i) $(f, g)=(aX+bY+e, cX+dY+e')$ where $ad-bc\neq 0$ or

(ii) $(f, g)=(X, Y+h(X))$ where $h(X)$ is an arbitrary polynomial.

By the theorem of Jung [J], (J.C. I) is equivalent to

(J.C. II) "If (1.1) is satisfied, (f, g) is an elementary transformation".

Let m=degree (g) and let g_m be the m-th homogeneous part of g. Among the various results about (J.C. I), the following is due to Abyankar [Ab]:
$g_m=0$ has at most 2 points in $P^1(K)$ if (1.1) is satisfied by f and g.
It is easy to prove that (J.C. II) is equivalent to (J.C. III) (See §4.):

(J.C. III) "If $g_m=0$ consists of two points, there is no polynomial f such that $J(f, g)=1$".

In this paper, we study the necessary conditions ("boundary obstruction") of the boundaries of the Newton polygon $N(g)$ for the existence of f such that $J(f, g)=1$. Unfortunately there exist polynomials which have no obstructions on the boundary. Our main results are in §6 (Theorem (6.3) etc.).

Received February 1, 1983