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KODAI MATH. J.
6 (1983), 193—197

COADJOINT EQUIVARIANCY OF MOMENTUM MAPPING

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1. A coadjoint orbit of a Lie group G is a G-homogeneous symplectic manifold with the inclusion map as the coadjoint equivariant momentum mapping. If (M, Ω) is a G-homogeneous symplectic manifold with the coadjoint equivariant momentum mapping, then the coadjoint equivariant momentum mapping gives a symplectic covering mapping onto a coadjoint orbit. And we have a result of B. Kostant which classifies the (simply connected) homogeneous symplectic manifolds with coadjoint equivariant momentum mappings (cf. [1], [2], [5]). An action with a fixed point may be considered to be antipodal with a homogeneous action. As explained in [4], we have some theorems concerning with the existence of (coadjoint equivariant) momentum mappings. In particular, the following three guarantee the coadjoint equivariancy of a momentum mapping:

(1) $H^2(\mathfrak{g}, R)=0$, where $\mathfrak{g}$ is the Lie algebra of G (cf. [4], [5], [6]),

(2) the symplectic form is an exact form of a G-invariant 1-form (cf. [1]), and

(3) G is a semidirect product of G_1 by G_2, where G_1 and G_2 have coadjoint equivariant momentum mappings, $H^1(\mathfrak{g}_1, R)=0$ and G_1 is connected (cf. [3], [4]).

In this paper, we give a condition for the coadjoint equivariancy of momentum mappings. The result is

PROPOSITION. *Let (M, Ω) be a connected symplectic manifold, and let G be a symplectic action on (M, Ω) with a momentum mapping. If the action G has a fixed point, then G has a coadjoint equivariant momentum mapping.*

2. Let (M, Ω) be a connected symplectic manifold, that is, M is a connected smooth manifold with a non-degenerate closed 2-form Ω. Ω induces a bundle isomorphism $\Omega^\flat: TM \to T^*M$ between the tangent bundle and the cotangent bundle of M defined by

$$\Omega^\flat(v)=v \lrcorner \Omega .$$

Denote the inverse of $\Omega^\flat$ by $\Omega^\#$. $\Omega^\#: T^*M \to TM$ is also a bundle isomorphism. Let $C^\infty(M)$ (resp. $\mathrm{aut}(M, \Omega)$) be the set of all real valued smooth functions (resp. Hamiltonian vector fields i.e., vector field X satisfying $L_X\Omega=0$) on M.

For each $f \in C^\infty(M)$, define $\beta(f)$ by

Received August 24, 1982