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ON THE EXISTENCE OF LONG PERIODIC ORBITS NEAR THE LIAPUNOV'S PERIODIC FAMILY IN GENERAL REASONANCE CASES

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1. Introduction.

In this note, we are concerned with the existence of periodic solutions for a Hamiltonian system with n degrees of freedom

$$\frac{dx_k}{dt}=\frac{\partial H}{\partial y_k}, \quad \frac{dy_k}{dt}=-\frac{\partial H}{\partial x_k} \qquad (k=1, \cdots, n). \tag{1.1}$$

We assume that the Hamiltonian function $H(x, y)=H(x_1, \cdots, x_n, y_1, \cdots, y_n)$ is smooth near the origin and vanishes with its first-order derivatives at the origin, which implies the origin is an equilibrium point. Here and in what follows, "smooth" means always C^∞. It is to be noted that the Hamiltonian function $H(x, y)$ is an integral for the system (1.1), i. e., the function $H(x, y)$ is constant along a solution curve for (1.1).

Let S denote the Hessian matrix of $H(x, y)$ at the origin and

$$J=\begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix},$$

where I is the $n\times n$ identity matrix. Then JS is the coefficient matrix of the linear terms of the vector field of (1.1) about the origin. As is well known [12], the eigenvalues of JS occur in pairs $\pm\lambda_1, \cdots, \pm\lambda_n$. If λ_1 is purely imaginary and none of the $n-1$ quotients $\lambda_k/\lambda_1 (k=2, \cdots, n)$ is an integer, a well-known theorem by Liapunov guarantees the existence of a one-parameter family of periodic solutions near the equilibrium (see [10] or section 16 of [12]). Recently many researches have been devoted to the study of the existence of periodic solutions in the cases when there exist integer-multiple eigenvalues of λ_1 among $\lambda_2, \cdots, \lambda_n$, i, e., resonance cases. In our previous paper [7], we considered an autonomous system possessing a nondegenerate integral under general resonance cases and established an existence theorem for long periodic solutions near an equilibrium. In this note, we restrict ourselves to the Hamiltonian system (1.1) and consider the same general resonance situation as in [7]. We then assume

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