H. UEDA
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AN EXTREMAL PROBLEM ASSOCIATED WITH THE SPREAD RELATION

BY HIDEHARU UEDA

0. Introduction. The notion of spread was introduced and investigated by Edrei [6], [7], who also conjectured the spread relation. This relation has now been proved by Baernstein [2] whose remarkable analysis rests on the introduction of a new function $T^*(z)$ $(z=re^{i\theta})$, closely related to Nevanlinna characteristic $T(r, f)$.

Let f be meromorphic and nonconstant. Suppose $\delta(\infty, f)>0$. Then it is suggested by Nevanlinna's theory that $|f(z)|$ must be "large" on a substantial portion of each circle $|z|=r$ when r is large. The spread relation provides a quantitative form of this statement.

To state this relation we require some notations. Let f be a meromorphic function of finite lower order μ. Fix a sequence $\{r_m\}$ of Pólya peaks of order μ of $f(z)$. Let $\Lambda(r)$ be a positive function with $\Lambda(r)=o(T(r, f))$ $(r\to\infty)$. Define the set of argument

$$E_\Lambda(r)=\{\theta : \log|f(re^{i\theta})|>\Lambda(r)\},$$

and let

$$\sigma_\Lambda(\infty)=\varliminf_{m\to\infty} \operatorname{meas} E_\Lambda(r_m)\,.$$

Then the spread of ∞ is defined by

$$\sigma(\infty)=\inf_\Lambda \sigma_\Lambda(\infty)\,,$$

where the "inf" is taken over all functions Λ satisfying $\Lambda(r)=o(T(r, f))$.

Spread relation:

$$\sigma(\infty)\geqq\min\left\{2\pi, \frac{4}{\mu}\sin^{-1}\sqrt{\frac{\delta(\infty, f)}{2}}\right\}. \tag{1}$$

(This inequality is best possible.) This makes it possible to solve the deficiency problem for functions with $1/2<\mu\leqq1$. (See [8].)

Baernstein's proof of the spread relation (1) is based on the properties of the function

$$T^*(re^{i\theta})=m^*(re^{i\theta})+N(r, f) \qquad (r>0,\ 0\leqq\theta\leqq\pi)\,, \tag{2}$$

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