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ASYMPTOTIC REDUCTION OF A CERTAIN BOUNDARY VALUE PROBLEM ARISING FROM THE DENSITY WAVE THEORY OF SPIRAL GALAXIES

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§1. Introduction.

The dynamical mechanism of the long term maintenance of the spiral structure observed in many disk shaped galaxies has been successfully explored through the density wave theory of the spiral galaxies by C. C. Lin and his collaborators. The mathematical formulation of this theory is based either on the steller dynamic or on the hydrodynamic approach. If we adopt the latter, the basic equation consists of the equation of continuity, the Euler's equations of motion, the equation of state and the Poisson equation. We assume that the basic equation has a steady axisymmetric solution, then we get equations of perturbations (2.1). In the context of the linear theory, there are three levels of approximations of the equations for analysis (private communication of Prof. C. C. Lin):

(i) an integro-differential system (2.3a), (2.3b), (2.3c) and (2.4), which is exact as the linearlized equations for the perturbations,

(ii) a third order differential system (2.7a) and (2.7b),

(iii) a second order differential equation (2.9).

Through numerical and asymptotic analyses of the second order differential equation, Lin and his school have arrived at a consistent interpretation of the plysical problems of the spiral galaxies, Lin and Lau [7]. Pannatoni and Lau [10] just begun the study of the integro-differential system by elaborate numerical analysis.

The purpose of this paper is to study the third order differential system of the second level of approximation by the asymptotic analysis and prove that it is asymptotically equivalent to the second order differential equation of the last level, in an appropriate region where the spiral structure prevails.

The program of this paper is as follows. In section 2, the basic equations and the equation of perturbation of each level of approximations are stated. In section 3, through the block diagonalization technique (Wasow [12], p. 133 ff.), the third order differential system is split into a second order and a first order differential equations. In section 4, we define the so-called admissible regions in which we construct asymptotic expansions of solutions of the third order differential system. And in the last section we consider connection formulas

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