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GENERIC SUBMANIFOLDS OF COMPLEX PROJECTIVE SPACES WITH PARALLEL MEAN CURVATURE VECTOR

Dedicated to Professor Shigeru Ishihara on his sixtieth birthday

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A submanifold M of a Kaehlerian manifold $\tilde{M}$ is called a generic submanifold (an anti-holomorphic submanifold) if the normal space $N_P(M)$ of M at any point $P \in M$ is always mapped into the tangent space $T_P(M)$ under the action of the almost complex structure tensor F of the ambient manifold, that is, $FN_P(M) \subset T_P(M)$ for all $P \in M$ (see [4], [9], [10] and [12]). The typical examples of generic submanifolds are real hypersurfaces of a Kaehlerian manifold. So many authors, for example, Kon [12], Okumura [9], Pak [9] and Yano [12] etc., have studied generic submanifolds of a Kaehlerian manifold by using the method of Riemannian fibre bundles and developed this method of Lawson [2], Maeda [5] or Okumura [8] extensively for real hypersurfaces.

In particular, two of the present authors [4] have studied generic submanifolds with parallel mean curvature vector of an even-dimensional Euclidean space under the condition that the f-structure induced on M is normal (see section 2).

The purpose of the present paper is to characterize generic submanifolds of complex projective space CP^m.

In §1, we investigate fundamental properties and structure equations for generic submanifolds immersed in a complex projective space CP^m. And we find the condition that the f-structure induced on M is normal.

In §2, we recall the theory of fibrations and some relations between the second fundamental tensor of M in CP^m and that of $\bar{M}=\tilde{\pi}^{-1}(M)$ in S^{2m+1}, and then establish some equations for the connections in the normal bundles of M and of $\bar{M}$, where $\tilde{\pi}$ is the projection induced from the Hopf-fibrations $S^1 \to S^{2m+1} \to CP^m$.

In the last §3, we characterize generic submanifolds of a complex projective space CP^m by the method of Riemannian fibration. In characterizing the submanifolds, we shall use the following theorem:

THEOREM A ([11]). *Let M be a complete n-dimensional submanifold of S^m with flat normal connection. If the second fundamental form of M is parallel, then M is a small sphere, a great sphere or a pythagorean product of a certain number of spheres. Moreover, if M is of essential codimension $m-n$, then M is a pythagorean product of the form*

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