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THE SPECTRUM OF THE LAPLACIAN FOR SOME 6-DIMENSIONAL K-SPACES

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1. Introduction.

Let (M, g) be a compact orientable Riemannian manifold with metric tensor g. By Δ we denote the Laplacian acting on differentiable functions on M. Then we have the spectrum

$$\mathrm{Spec}(M, g)=\{0\geqq\lambda_1\geqq\lambda_2\geqq\cdots>-\infty\}$$

where each eigenvalue is repeated as many time as its multiplicity indicates. The spectrum $\mathrm{Spec}(M, g)$ exerts an influence on the geometry of (M, g). It is interesting to see the relation of $\mathrm{Spec}(M, g)$ on the geometry of (M, g). For the study of this, M. Berger and T. Sakai used the coefficients of the asymptotic expansion of Minakshisundaram-Pleijel. In [6], after a long calculation, Sakai obtained the following

THEOREM A. *Let (M, g) and (M', g') be compact connected orientable Einstein manifolds with dimension $M=6$. We assume that $\chi(M)=\chi(M')$ and $\mathrm{Spec}(M, g)=\mathrm{Spec}(M', g')$ hold where $\chi(M)$ denotes the Euler-Poincaré characteristic of M. Then (M, g) is locally symmetric if and only if (M', g') is locally symmetric.*

In the present paper, we shall prove the following

THEOREM B. *Let (M, g, J) and (M', g', J') be 6-dimensional complete, connected K-spaces which are non-Kählerian. We assume that $\chi(M)=\chi(M')$ and $\mathrm{Spec}(M, g)=\mathrm{Spec}(M', g')$. Then (M, g) is Riemannian locally 3-symmetric if and only if (M', g') is Riemannian locally 3-symmetric.*

It is well-known that the 6-dimensional non-Kähler K-space (M, g, J) is an Einstein manifold with positive scalar curvature [5]. Therefore M is compact by Myers' theorem. The study of Riemannian 3-symmetric space has been done by A. Gray [4]. We shall give some definitions and preliminary facts on Riemannian 3-symmetric spaces in § 2. Particularly we shall show the relationship between Riemannian 3-symmetric spaces and homogeneous K-spaces. In § 3, we shall prove Theorem B by slight modification of the proof of Theorem A.

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