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KODAI MATH. J.
4 (1981), 1—27

ALMOST CONTACT STRUCTURES AND CURVATURE TENSORS

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Abstract

We determine an orthogonal decomposition of the vector space of some curvature tensors on a co-Hermitian real vector space, in irreducible components with respect to the natural induced representation of $\mathcal{U}(n)\times 1$. One of the components is used to introduce a Bochner curvature tensor on a class of almost co-Hermitian manifolds (or almost contact metric manifolds), called $C(\alpha)$-manifolds, containing e.g. co-Kählerian, Sasakian and Kenmotsu manifolds. Other applications of the decomposition are given.

1. Introduction.

In his study on Betti numbers of Kähler manifolds, S. Bochner introduced a tensor which had to take over in his theory the role of the Weyl tensor on Riemannian manifolds. More precisely, a conformally flat manifold was considered as an extension of a real space form. So, a Bochner flat Kähler manifold had to be an extension in the same sense of a complex space form.

The Weyl tensor is well known as a conformal invariant of Riemannian manifolds but the Bochner tensor on Kähler manifolds was defined in a completely formal way. Several attempts were made to find a geometrical interpretation.

Besides Kähler manifolds and the more general almost Hermitian manifolds, one has also studied classes of odd-dimensional manifolds with additional structures, namely the almost co-Hermitian manifolds or almost contact metric manifolds; in particular Sasakian, co-Kählerian and Kenmotsu manifolds.

A natural problem arises here: Is it possible to construct a "Bochner curvature tensor" for these classes of manifolds? A nice way to introduce this tensor is to use decomposition theory of spaces of curvature tensors. Singer and Thorpe obtained in this way the Weyl tensor [20] (see also [17]) and the Bochner tensor was derived with the Hermitian version of this decomposition in [16], [21]. Proceeding in the same way, the second author defined a Bochner tensor for AH_3-manifolds (a class of almost Hermitian manifolds containing e.g. the nearly Kähler manifolds) [24], [27]. The general case is given completely in [34].

In this paper we define with a decomposition theory a Bochner curvature

Received October, 18, 1979