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COMPLEX ALMOST CONTACT MANIFOLDS

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§ 1. Introduction.

A complex contact manifold is a complex manifold of odd dimensions $2m+1$ ($\geqq 3$) covered by an open covering $\mathcal{A}=\{O, O', \cdots\}$ consisting of coordinate neighborhoods in such a way that

1) In each $O\in\mathcal{A}$ there is a holomorphic 1-form w satisfying $w\wedge(dw)^m\neq 0$ at every point of O;

2) If $O\cap O'\neq\phi$ $(O, O'\in\mathcal{A})$, there is a non-vanshing holomorphic function λ in $O\cap O'$ such that $w'=\lambda w$ in $O\cap O'$, where w' is the holomorphic 1-form given in O' (See Kobayashi [3]).

In a previous paper [2] we have studied complex contact structure $\{(O, w)|O\in\mathcal{A}\}$ which are induced by fiberings of manifolds with (real) normal contact 3-structure and obtained the induced (local) tensor field G of type (1, 1) in each $O\in\mathcal{A}$ such that $G^2=-I+w\otimes W$, $w\circ G=0$, where W is the associated vector field of w. The local structures $\{(O, G, w, W)|O\in\mathcal{A}\}$ are very useful to study curvature properties in the same way as in the real case (See Gray [1]. and Sasaki [4]). In the present paper we first define a system of local structures $\{(O, u, G)|O\in\mathcal{A}\}$ which will be called a complex almost contact structure and next show that such a structure induces a complex contact structure defined by Kobayashi [3], when it satisfies a suitable condition, i.e., to be normal.

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§ 2. Complex almost contact structure.

Let M be a complex manifold with complex structure F and Hermitian metric g and be covered by an open covering $\mathcal{A}=\{O, O', \cdots\}$ consisting of coordinate neighborhoods. Then M is called a *Complex almost contact manifold* if the following conditions **1**) and **2**) are satisfied:

1) In each $O\in\mathcal{A}$ there are given a 1-form u and a tensor field G of type (1, 1) such that*)

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*) Functions vector fields, tensor fields and geometric objects we consider are assumed to be differentiable and of class C^∞, otherwise stated. Throughout this paper, X, Y and Z denote arbitrary vector fields in M.