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ON SUBORDINATION OF SUBHARMONIC FUNCTIONS

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Introduction. In the present paper we are concerned with analytic maps from a Riemann surface into another which preserve the least harmonic majorant of a subharmonic function.

Let R denote an open Riemann surface. Let $S(R)$ be the class of all functions subharmonic on R which admit harmonic majorants on R and $S^+(R)=\{f\in S(R): f \text{ is bounded below on } R\}$. We denote by $\hat{f}$ the least harmonic majorant of f for any $f\in S(R)$. Let R_j $(j=1, 2)$ be open Riemann surfaces and ϕ be an analytic map from R_1 into R_2. Littlewood's subordination theorem (see [3, p. 10]) shows that $f\circ\phi\in S(R_1)$ and

$$\hat{f}\circ\phi\geqq\widehat{f\circ\phi} \tag{1}$$

on R_1 for any $f\in S(R_2)$. In this paper we deal with the problem when equality holds in (1).

In the case where $R\in O_G$, it is well known that there exist no positive superharmonic functions but the constants on R (see for example [1, p. 204]). Therefore we easily see that $\hat{f}-f\equiv 0$ for any $f\in S(R)$, which means that $S(R)$ reduces to the harmonic functions. It is easily verified that if $R_1\in O_G$ and $R_2\notin O_G$ there exist no nonconstant analytic maps from R_1 into R_2. Hence, if one of R_j $(j=1, 2)$ is of class O_G, equality always holds in (1) for any $f\in S(R_2)$.

From now on we assume that $R_j\notin O_G$ for $j=1, 2$. Let $G_j(z, t)$ denote the Green's function of R_j with pole at t. Following Heins [4], we say that ϕ is of *type B1* when $G_2(\phi(z), t)$ majorates no positive bounded harmonic functions for some $t\in R_2$, or equivalently for every $t\in R_2$ (see Theorem 4.1 of [4, p. 446]), and we say that ϕ is of *type $B1_1$* when $G_2(\phi(z), t)$ majorates no positive harmonic functions for every $t\in R_2$. Let U denote the open unit disc and π_j be a universal covering map of R_j. By applying the monodromy theorem, we can define an analytic function ψ in U which is bounded by 1 such that

$$\phi\circ\pi_1=\pi_2\circ\psi\,. \tag{2}$$

An *inner function* is any function ψ analytic in U with the properties $|\psi(z)|\leqq 1$ in U and $|\psi^*(e^{i\theta})|=1$ a.e. on ∂U, where ψ^* denotes the Fatou's boundary function of ψ.

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