H. IMAI
KODAI MATH. J.
3 (1980), 56—58

ON THE RATIONAL POINTS OF SOME JACOBIAN VARIETIES OVER LARGE ALGEBRAIC NUMBER FIELDS

BY HIDEO IMAI

In this note we shall prove the following: Let X be a hyperelliptic curve defined over the rational number field $\boldsymbol{Q}$ and let J be its Jacobian variety. Let L be the field generated by all square roots of rational integers over $\boldsymbol{Q}$. Then the group of L-rational points $J(L)$ has an infinite rank over the rational integer ring $\boldsymbol{Z}$.

In Frey and Jarden [1], the following is conjectured: Let A be an abelian variety defined over $\boldsymbol{Q}$ and $\boldsymbol{Q}_{ab}$ the maximal abelian extension of $\boldsymbol{Q}$. Then does the group $A(\boldsymbol{Q}_{ab})$ have an infinite rank over $\boldsymbol{Z}$? Our result supports this conjecture partially.

1. Let X be a hyperelliptic curve defined by the equation (in the affine form) $y^2=f(x)$, where $f(x)$ is a monic separable polynomial of degree $2g+1$ with coefficients in $\boldsymbol{Z}$. Let $P_0=(\infty, \infty)$ be the point at infinity on X, which is rational over $\boldsymbol{Q}$. Let $z=x^g/y$ be a local uniformizing parameter at P_0. Let $\omega_i=x^{i-1}dx/y$ $(i=1, 2, \cdots, g)$ be the canonical base of the space of differential forms of the first kind on X. Writing these ω_i in terms of z and integrating ω_i formally, we get power series $\Psi_i(z)\in\boldsymbol{Q}[[z]]$ such that $\Psi_i(0)=(0)$ and $\omega_i=d\Psi_i$.

LEMMA 1.
$$\Psi_i(z)=\frac{-2}{2g-2i+1}z^{2g-2i+1}+\sum_{n>g-i}\frac{c_n^{(i)}}{2n+1}z^{2n+1}\quad with\quad c_n^{(i)}\in\boldsymbol{Z}.$$

Proof. It is easily proved by direct computation. We outline the proof. Differentiating $z=x^g/y$ with respect to x, we have
$$dz=(gx^{g-1}-x^g f'(x)/2f(x))dx/y\,.$$
Hence we have
$$\Psi_i'(z)=1/gx^{g-i}(1-xf'(x)/2gf(x))\,.$$
We write $z=x^g/\sqrt{f(x)}$ and expand the above equation in terms of $t=1/x$. Let $\Psi_i(z)=\sum_{n=1}^{\infty}a_n z^n$ and let $h(1/x)=f(x)/x^{2g+1}-1$. After some computations we get

Received January 17, 1979