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COEFFICIENTS OF INVERSES OF UNIVALENT FUNCTIONS WITH QUASICONFORMAL EXTENSIONS

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§ 1. Introduction.

Let Σ' denote the family of univalent functions

$$g(z)=z+\sum_{n=1}^{\infty}\frac{b_n}{z^n}$$

in $\tilde{\Delta}=\{z: 1<|z|<\infty\}$. For $0\leqq k<1$ let Σ'_k be the family of functions in Σ' that admit k-quasiconformal extensions to $\bar{\Delta}=\{z:|z|\leqq 1\}$. That is, each $g\in\Sigma'_k$ has a homeomorphic extension to $\bar{\Delta}$, that is absolutely continuous on a.e. horizontal and vertical line in $\bar{\Delta}$ and satisfies

$$|g_{\bar{z}}|\leqq k|g_z| \qquad \text{a.e. in } \bar{\Delta}.$$

If $k=0$, then g is an entire univalent function. Consequently, Σ'_0 contains only the identity function. As $k\to 1$, the families Σ'_k are dense in Σ', and we therefore define $\Sigma'_1=\Sigma'$. Since $\Sigma'_{k_1}\subset\Sigma'_{k_2}$ for $k_1<k_2$, the families Σ'_k interpolate in a monotonic fashion from the identity function to the family Σ'.

R. Kühnau [2] and O. Lehto [5] have obtained the sharp coefficient estimates

$$|b_1|\leqq k \quad \text{and} \quad |b_2|\leqq (2/3)k \tag{1}$$

for functions $g\in\Sigma'_k$. In this article we shall study the coefficients of their inverse functions.

That is, if G is the inverse of a function g in Σ'_k, i.e., $G=g^{-1}$, then G has an expansion

$$G(w)=w+\sum_{n=1}^{\infty}\frac{B_n}{w^n}$$

in some neighborhood of $w=\infty$. Since $B_1=-b_1$ and $B_2=-b_2$, the sharp estimates

$$|B_1|\leqq k \quad \text{and} \quad |B_2|\leqq (2/3)k \tag{2}$$

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